Breaking waves: Energetics, Bubbles & Droplets

Wouter Mostert

Stephane Popinet

Luc Deike

Basilisk Users' Meeting

Paris, France – 17-19 June 2019

Motivation & Outline

- Breaking action transfers energy, momentum between ocean, atmosphere

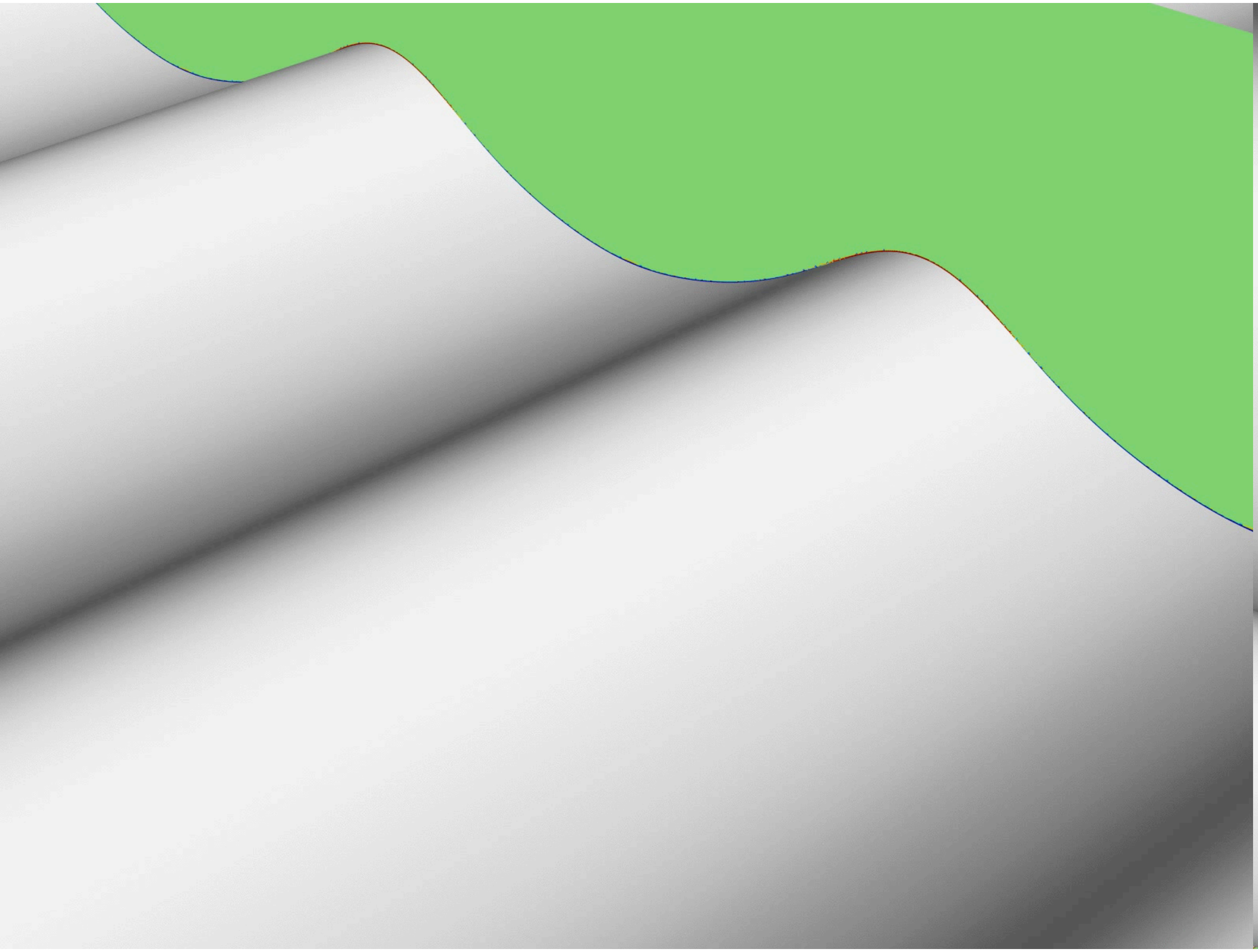
- Deep water energetics
- Dimensionality of transition to turbulence
- Bubbles and spray
- Shallow water breaker energetics

Waves off Atlantic Beach, NC – Thursday Sep 13 2018
Credit: Travis Long, AP (http://www.startribune.com/time-nearly-up-fierce-hurricane-florence-aims-at-southeast/493121431/#1)

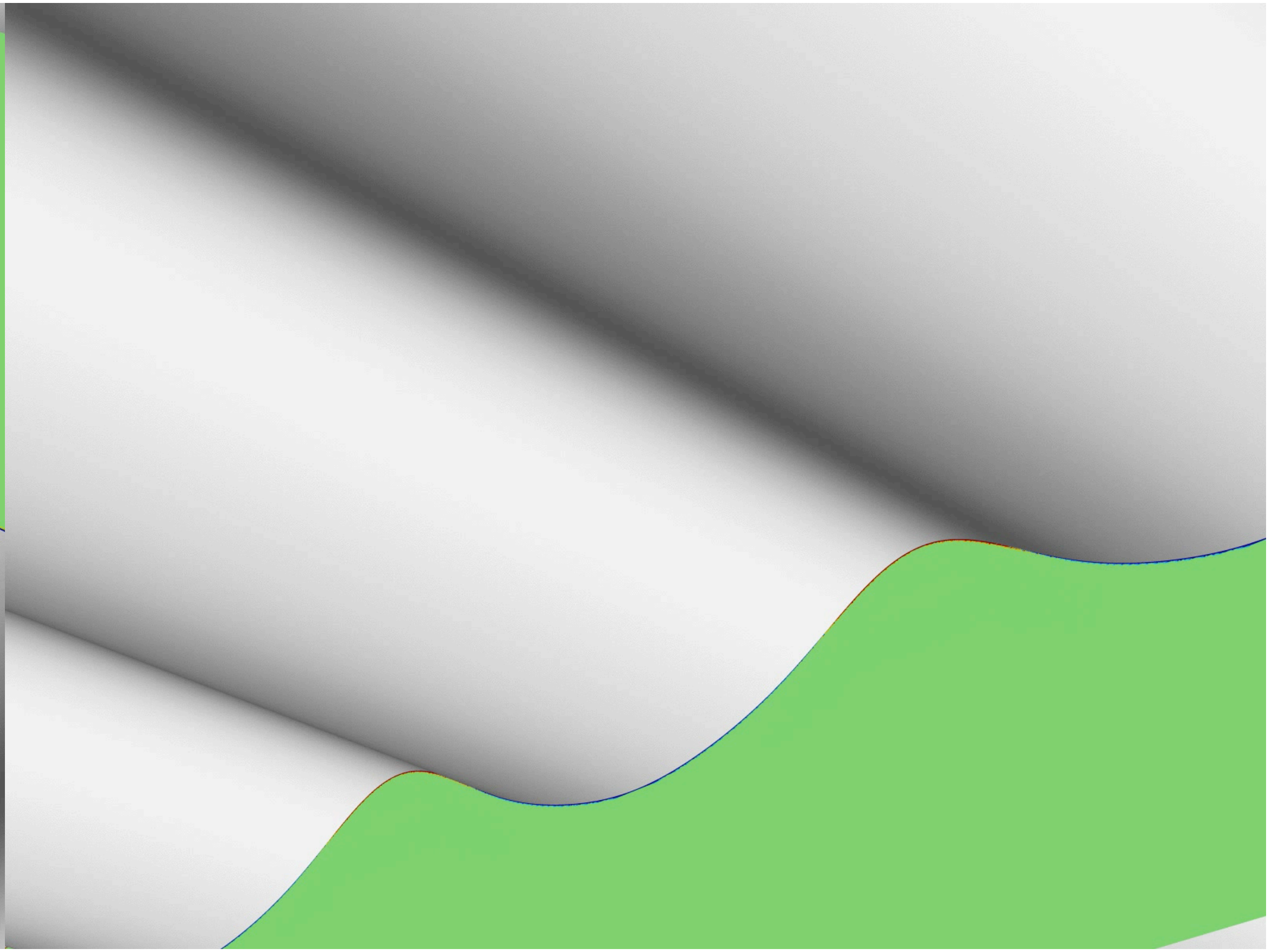
PRINCETON UNIVERSITY

Deep water breakers

Deep water breakers



Plunging breaker – view from above

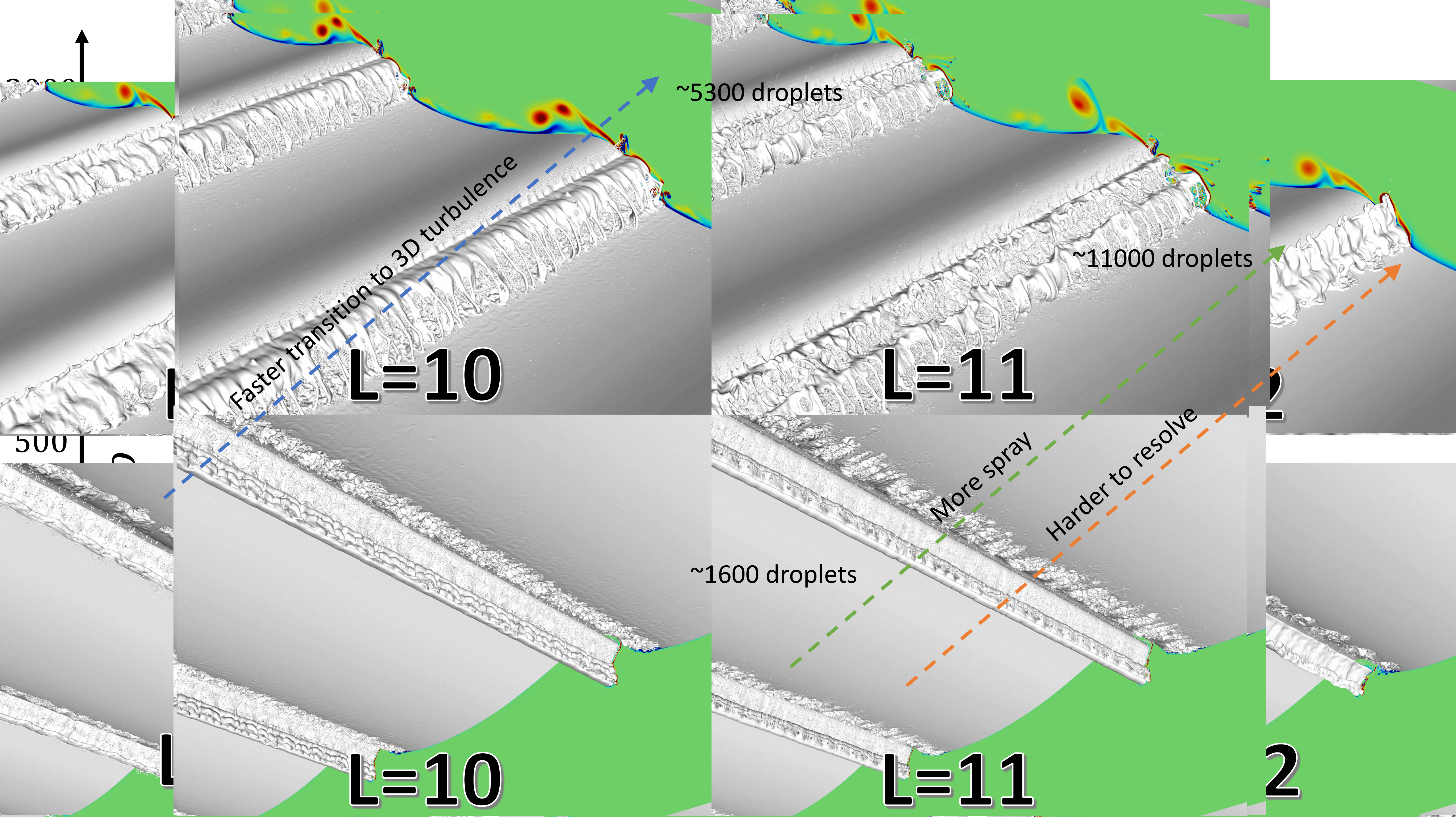


View from below

Computed on Extreme Science and Engineering Discovery Environment (XSEDE), supported by National Science Foundation grant number ACI-1548562.

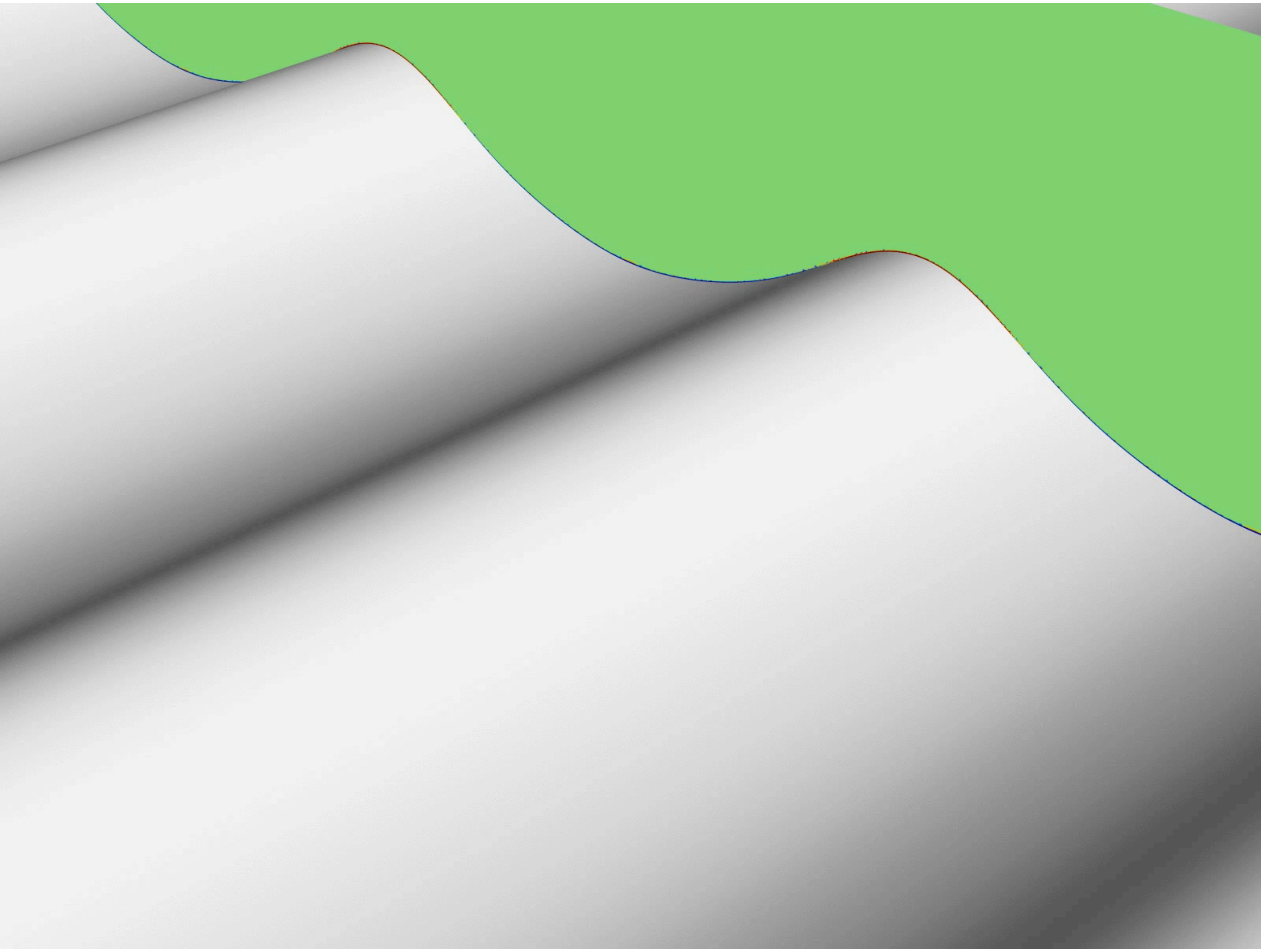
PRINCETON UNIVERSITY

Mostert, Popinet & Deike (2019) – under preparation



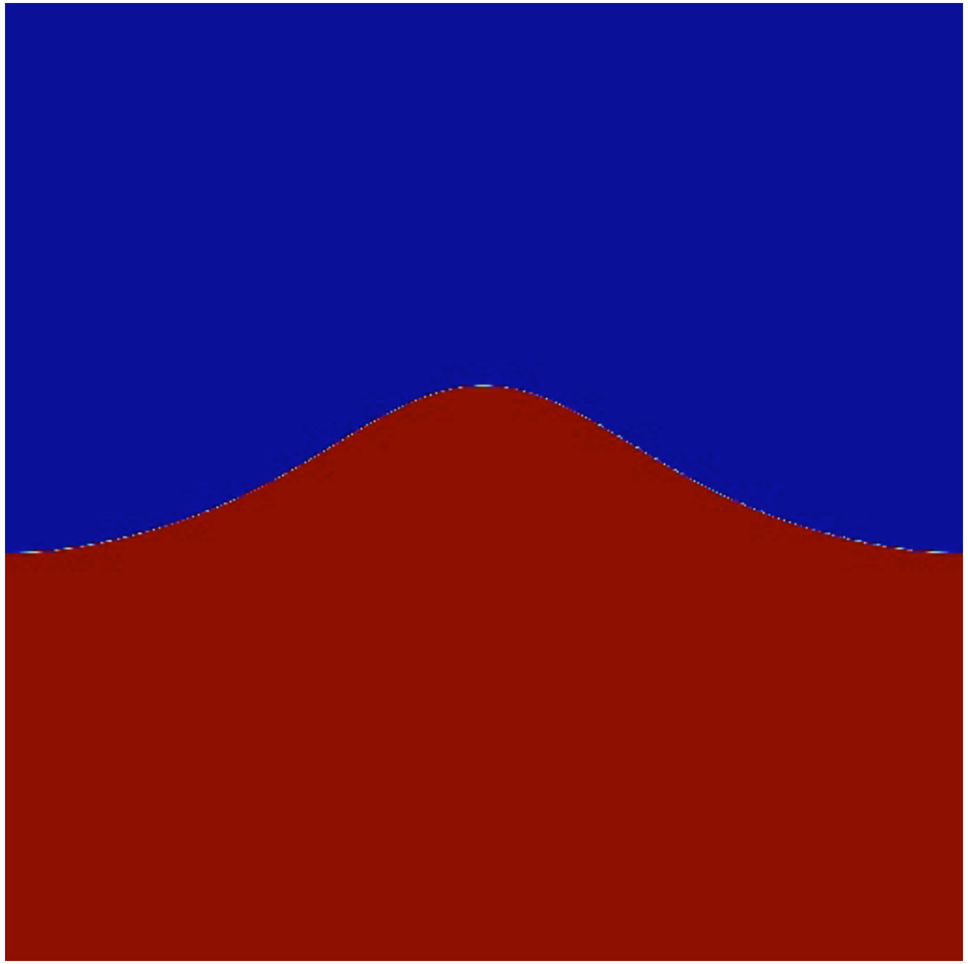
~5300 droplets
Faster transition to 3D turbulence
L=10
L=11
~11000 droplets
500
More spray
Harder to resolve
~1600 droplets
L=10
L=11
2

Deep water breakers



3D simulation – isosurface of water volume fraction

Computed on Extreme Science and Engineering Discovery Environment (XSEDE), supported by National Science Foundation grant number ACI-1548562.



2D Simulation – volume fraction of water

Mostert, Popinet & Deike (2019) – under preparation

PRINCETON UNIVERSITY

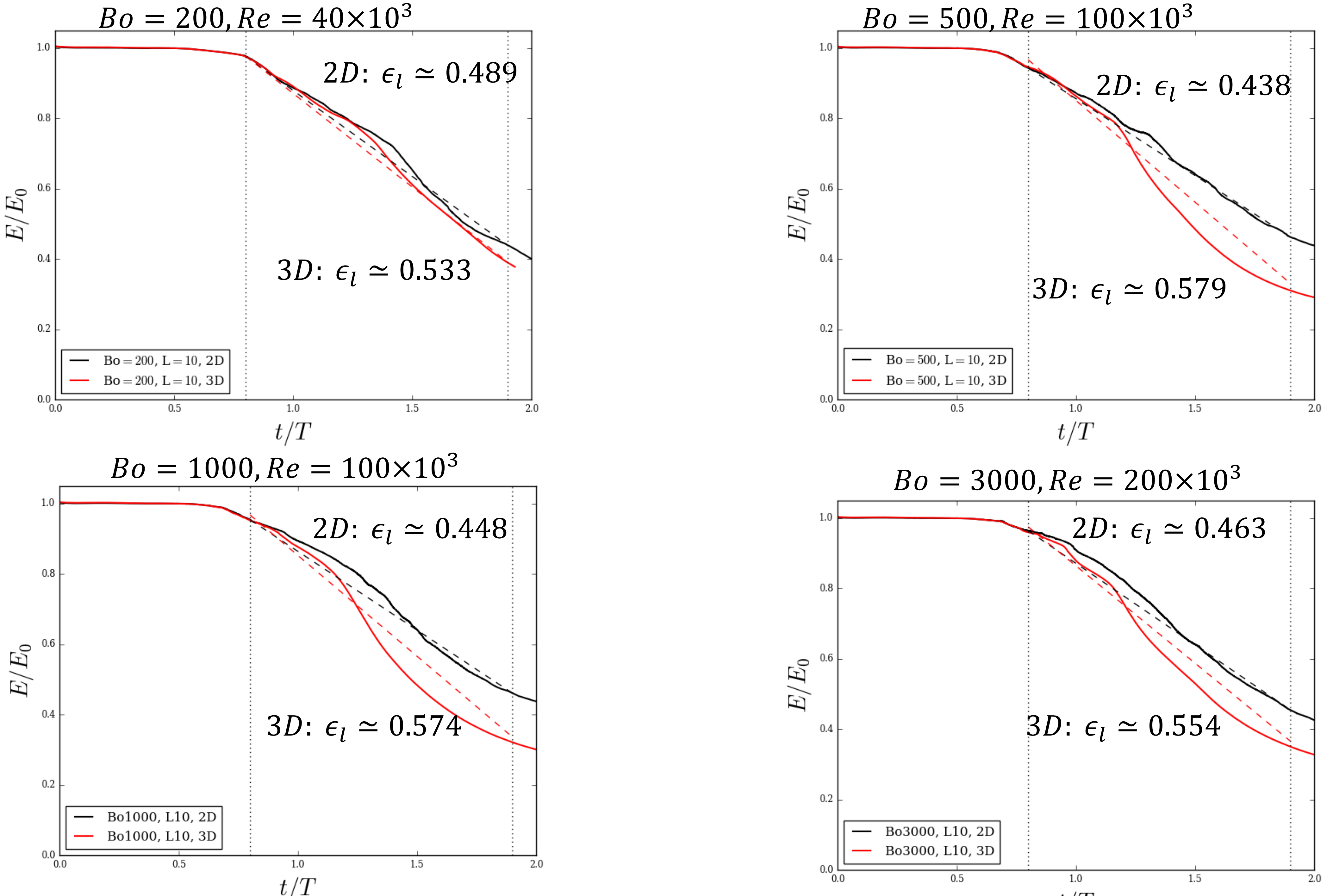
Dimensionality in transition to turbulence

- Energy dissipation in 2D breaking waves approximates the full 3D case (Deike et al. 2015, 2016; Iafrati 2009; Derakhti and Kirby 2014; Derakhti et al 2016)

Why & how is this so?

- Compare 2D and 3D simulations in deep water

PRINCETON UNIVERSITY



$Bo = 200, Re = 40\times10^3$
$2D$: $\epsilon_l \simeq 0.489$
$3D$: $\epsilon_l \simeq 0.533$
Bo = 200, L = 10, 2D
Bo = 200, L = 10, 3D
$Bo = 500, Re = 100\times10^3$
$2D$: $\epsilon_l \simeq 0.438$
$3D$: $\epsilon_l \simeq 0.579$
Bo = 500, L = 10, 2D
Bo = 500, L = 10, 3D
$Bo = 1000, Re = 100\times10^3$
$2D$: $\epsilon_l \simeq 0.448$
$3D$: $\epsilon_l \simeq 0.574$
Bo1000, L10, 2D
Bo1000, L10, 3D
$Bo = 3000, Re = 200\times10^3$
$2D$: $\epsilon_l \simeq 0.463$
$3D$: $\epsilon_l \simeq 0.554$
Bo3000, L10, 2D
Bo3000, L10, 3D
E/E_0
t/T

Deformation tensor – 2D & 3D

- Energy dissipation rate,

$$\dot{E} = 2\nu \int_V \sum_i \sum_j D_{ij}^2 dV$$
$$= 2\nu \sum_i \sum_j \int_V D_{ij}^2 dV$$
$$= \sum_i \sum_j E_{ij}$$

where

$$E_{ij} = 2\nu \int_V D_{ij}^2 dV = \frac{\nu}{2} \int_V \left[\partial_i u_j + \partial_j u_i\right]^2 dV$$

In 3D, the components are:

$$\begin{matrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{matrix}$$

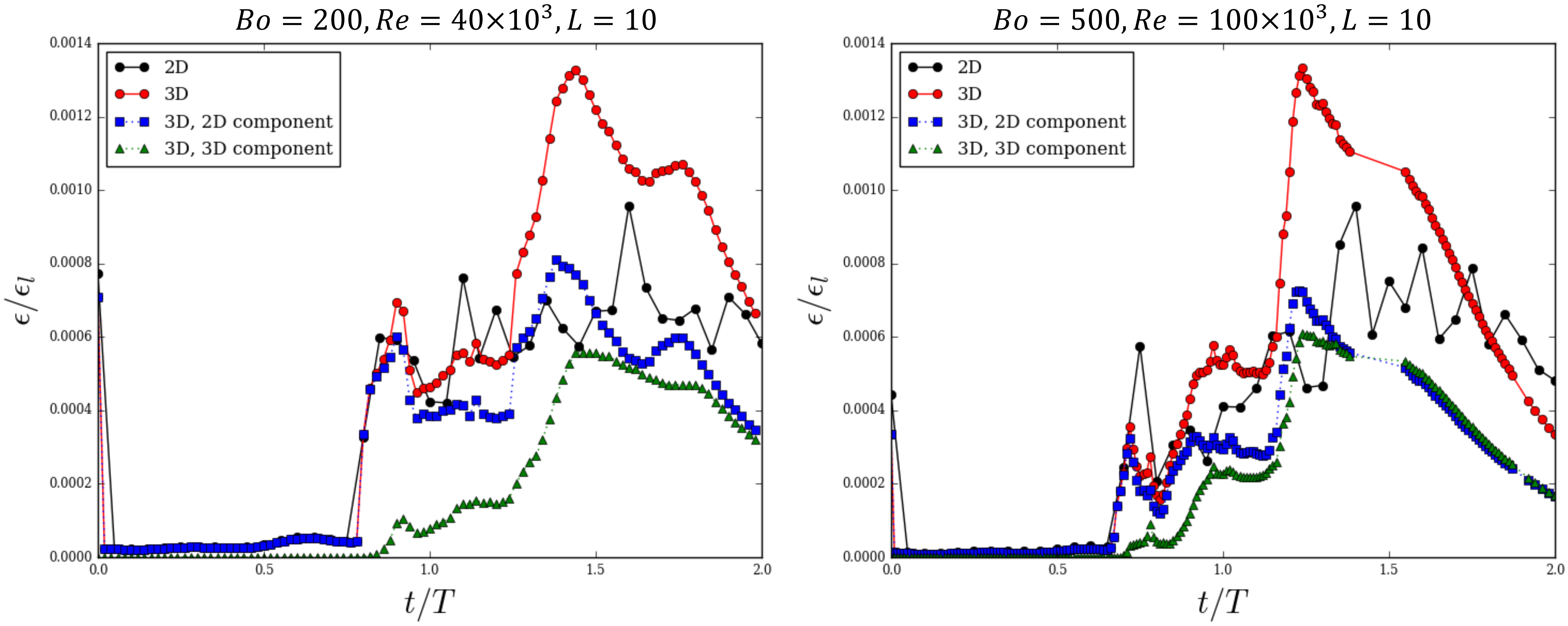
2D contributions

3D contributions

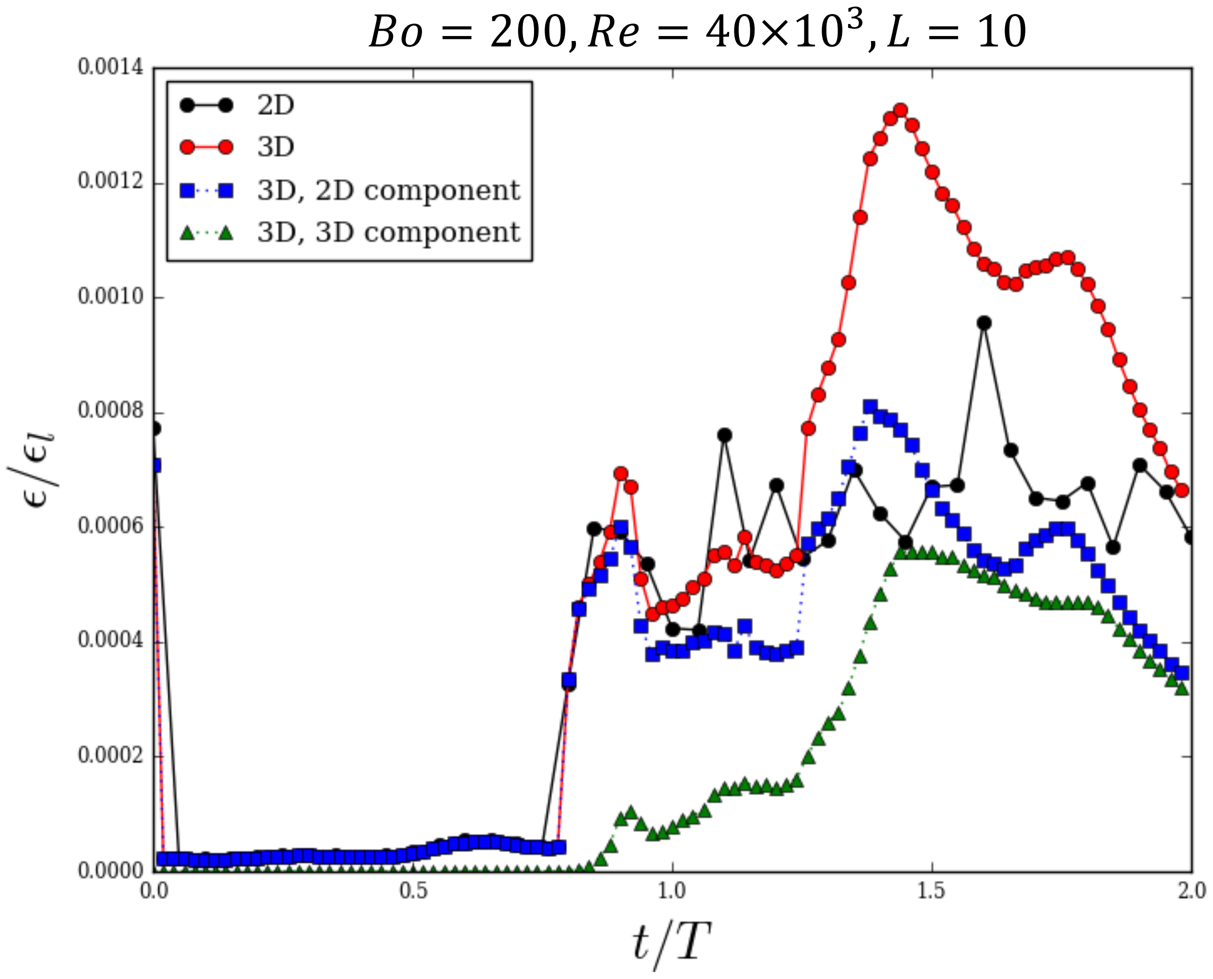
$$\dot{E} = \left(\dot{E}_{2D} + \dot{E}_{3D}\right)$$
$$\dot{E}_{2D} = \sum_i^2 \sum_j^2 E_{ij}$$
$$\dot{E}_{3D} = \sum_i^2 E_{i3} + \sum_j^3 E_{3j}$$

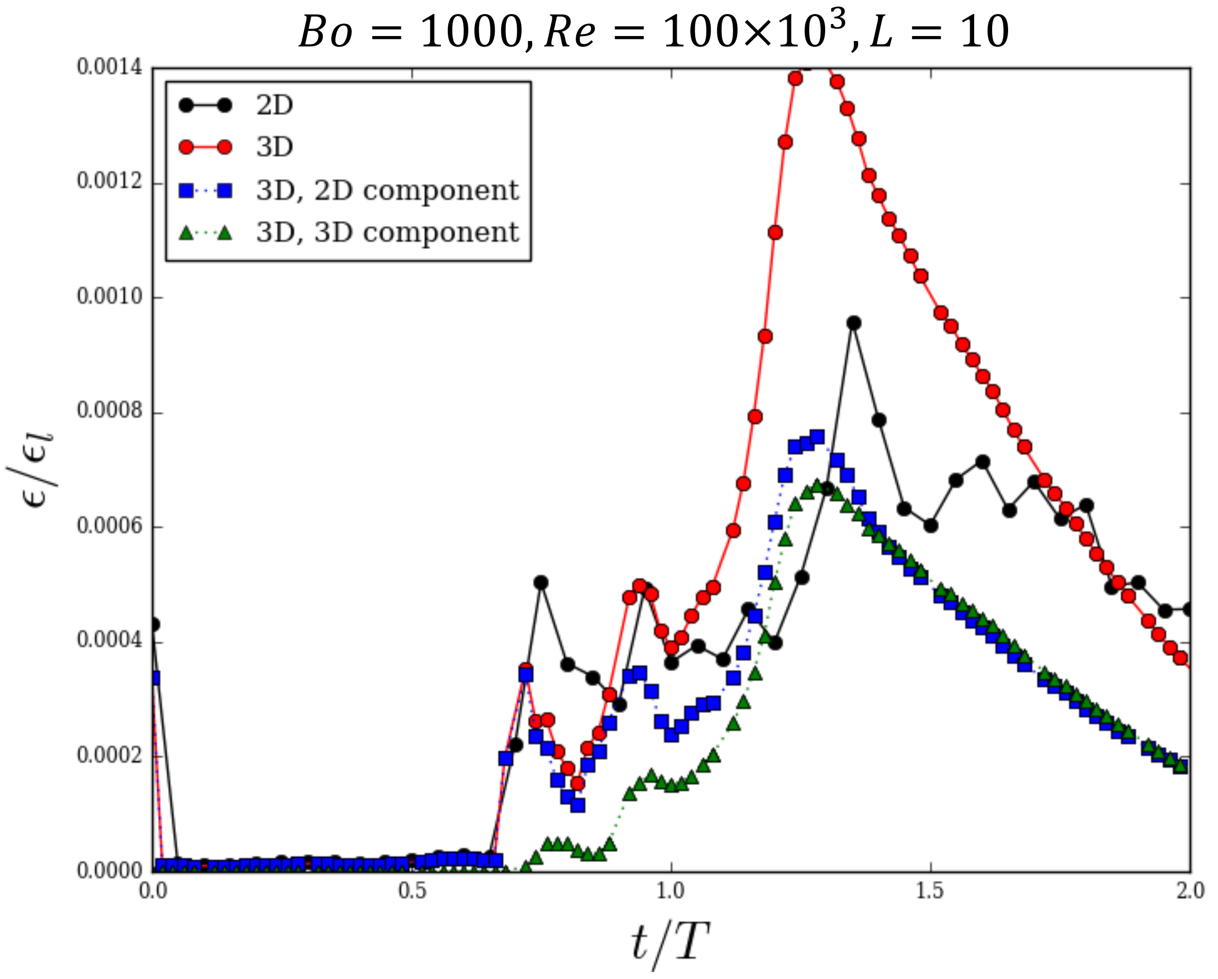
PRINCETON UNIVERSITY

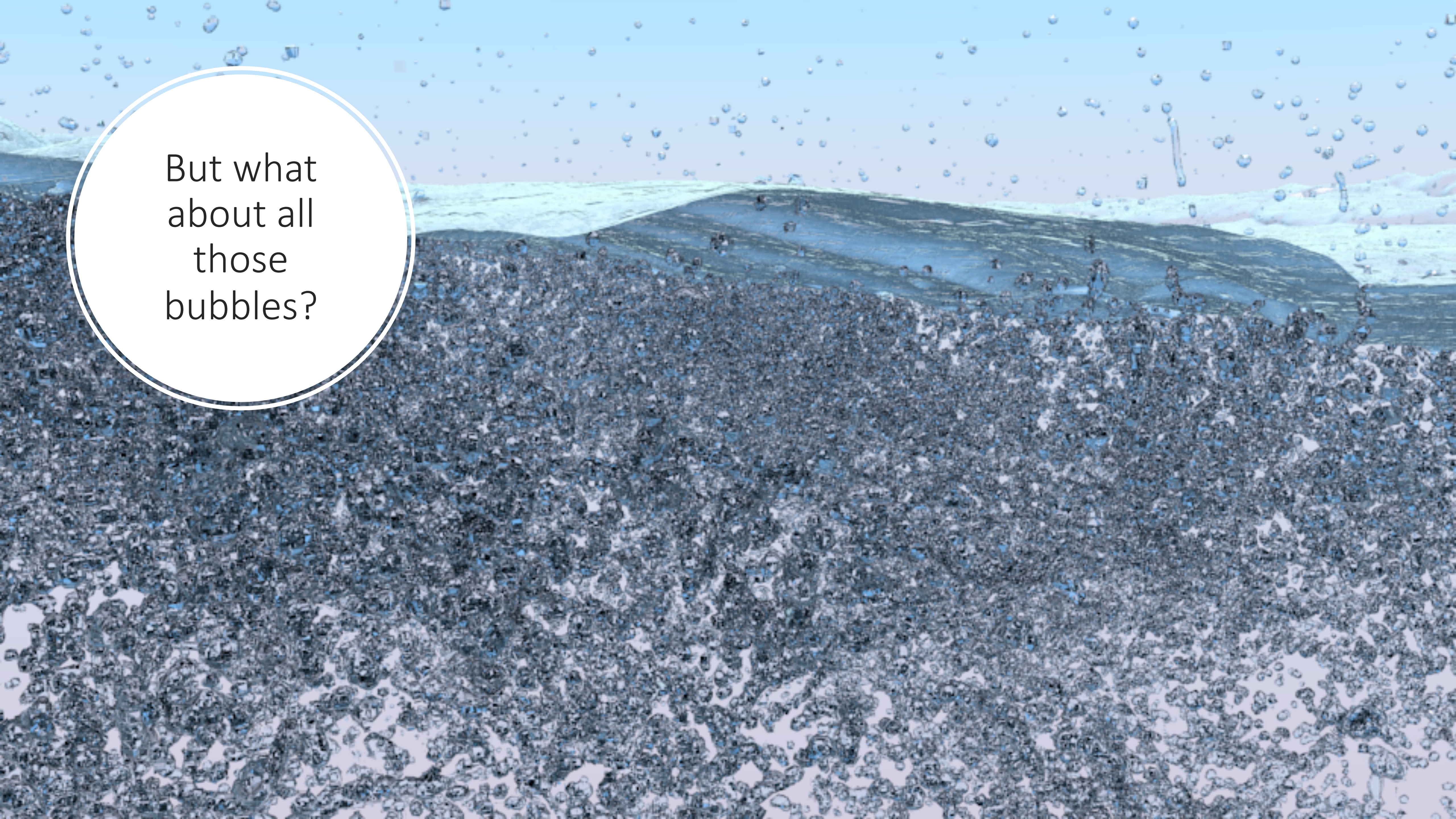
- Instantaneous 2D rate matches 3D up to t/T = 0.8
- 2D component of 3D dissipation roughly matches complete 2D to t/T=0.8
- At end 2D/3D components of 3D dissipation equal, less than complete 2D



- Instantaneous 2D rate matches 3D up to t/T = 0.8
- 2D component of 3D dissipation roughly matches complete 2D to t/T=0.8
- At end 2D/3D components of 3D dissipation equal, less than complete 2D

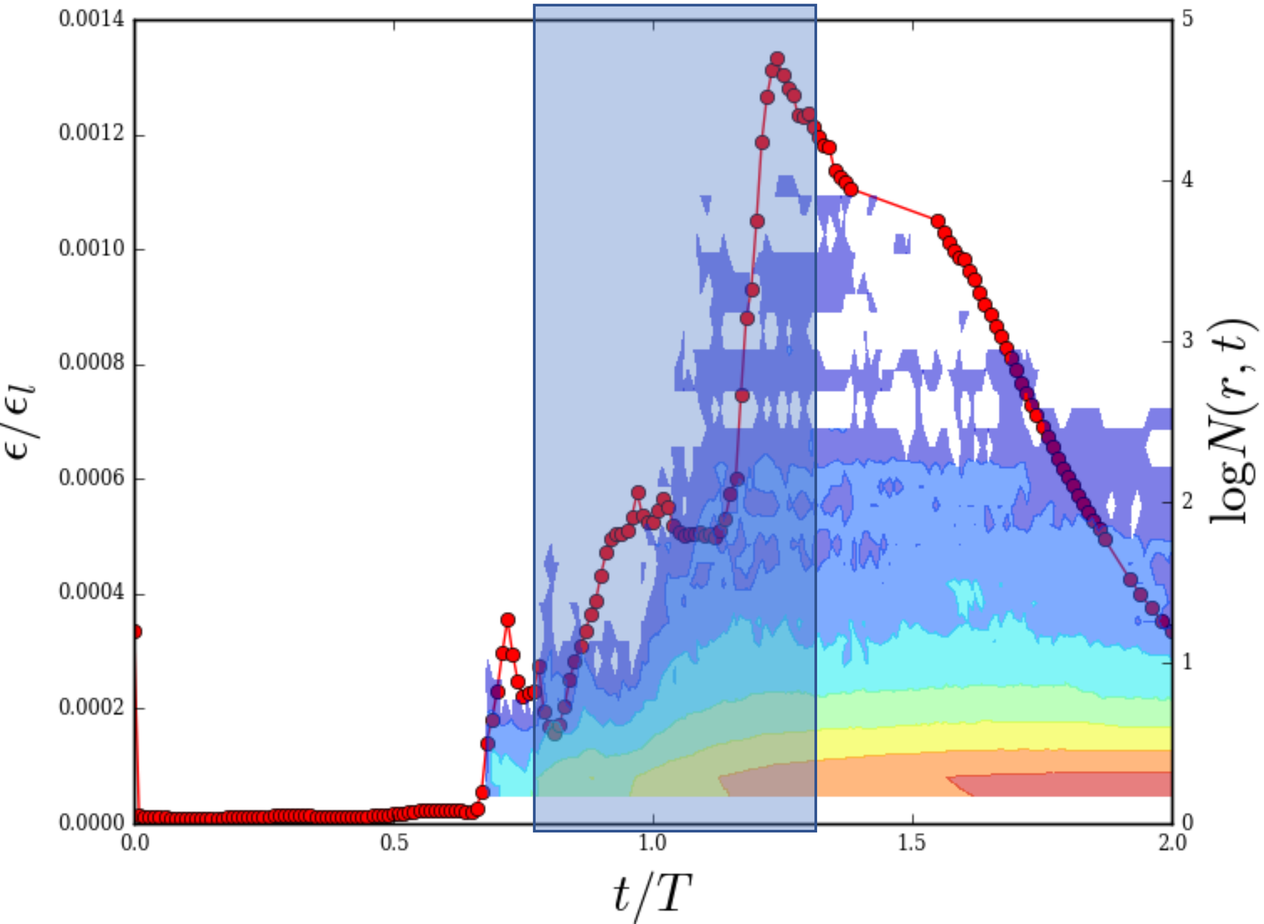


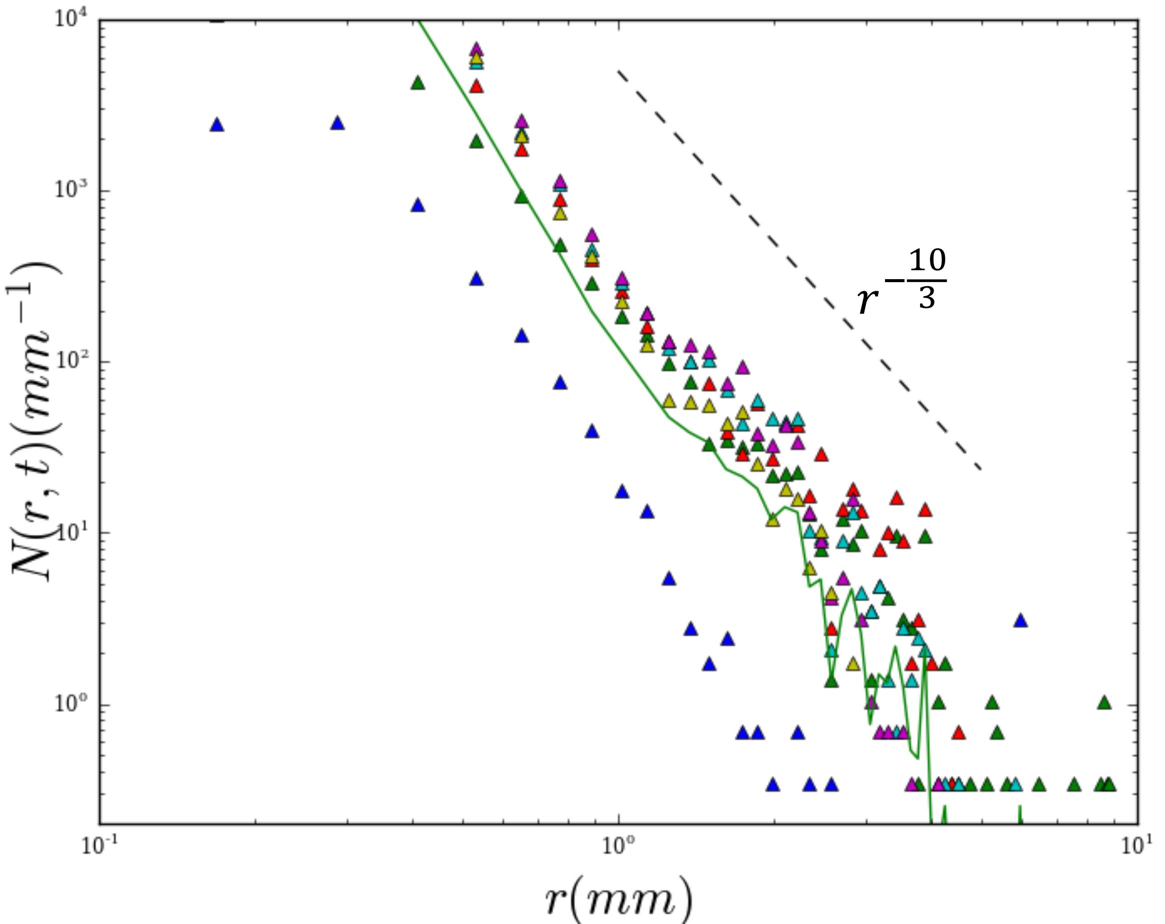



But what about all those bubbles?

$Bo = 500, Re = 100 \times 10^3$

- Bubble count roughly follows dissipation rate in time
- Bubble size distribution matches -10/3 power-law (Garrett et al 2000)

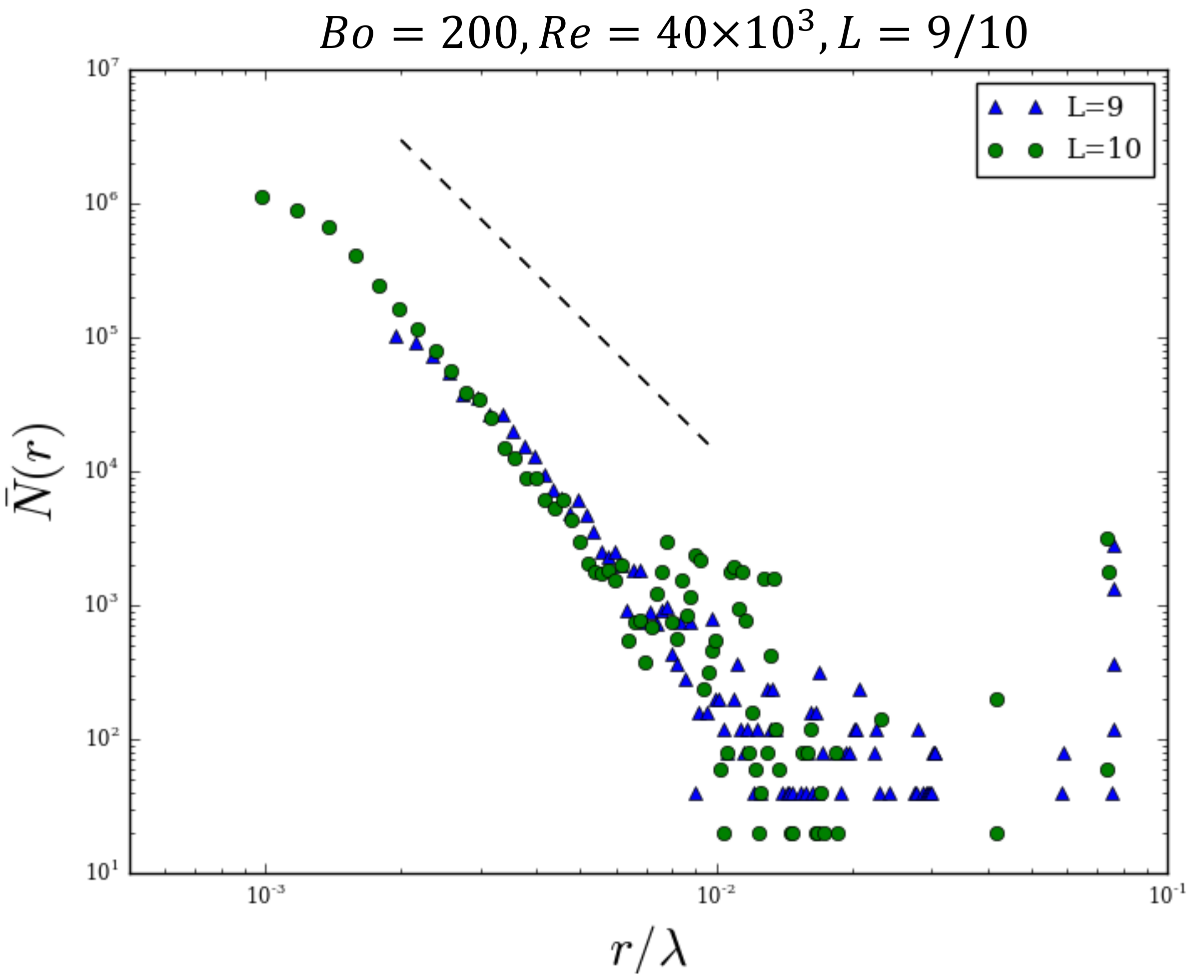


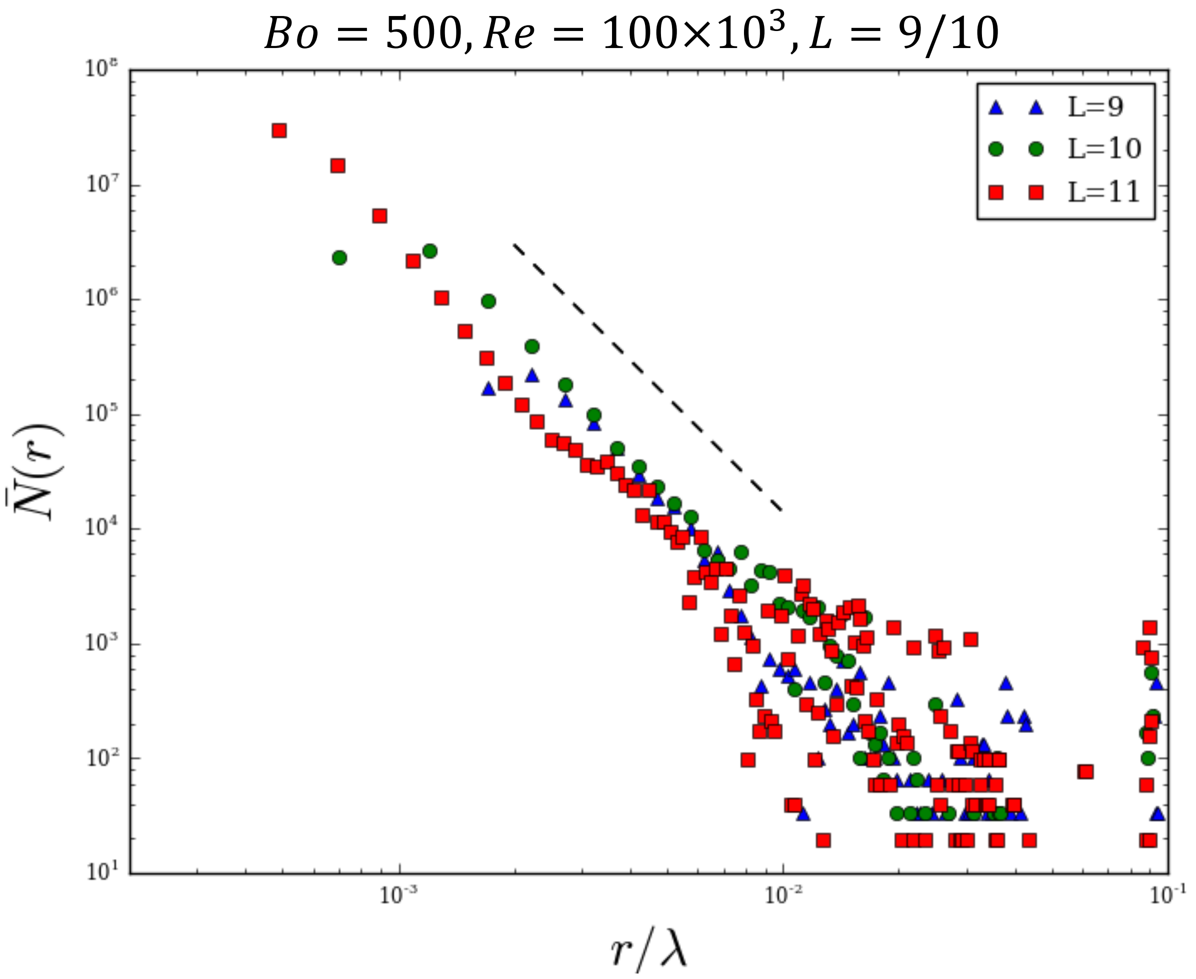


Bubble size distribution

- Averaged $\frac{t}{T} = [0.8, 1.3]$
- Grid comparison L9/10/11

- Bubble size distribution matches -10/3 power-law (Garrett et al 2000)



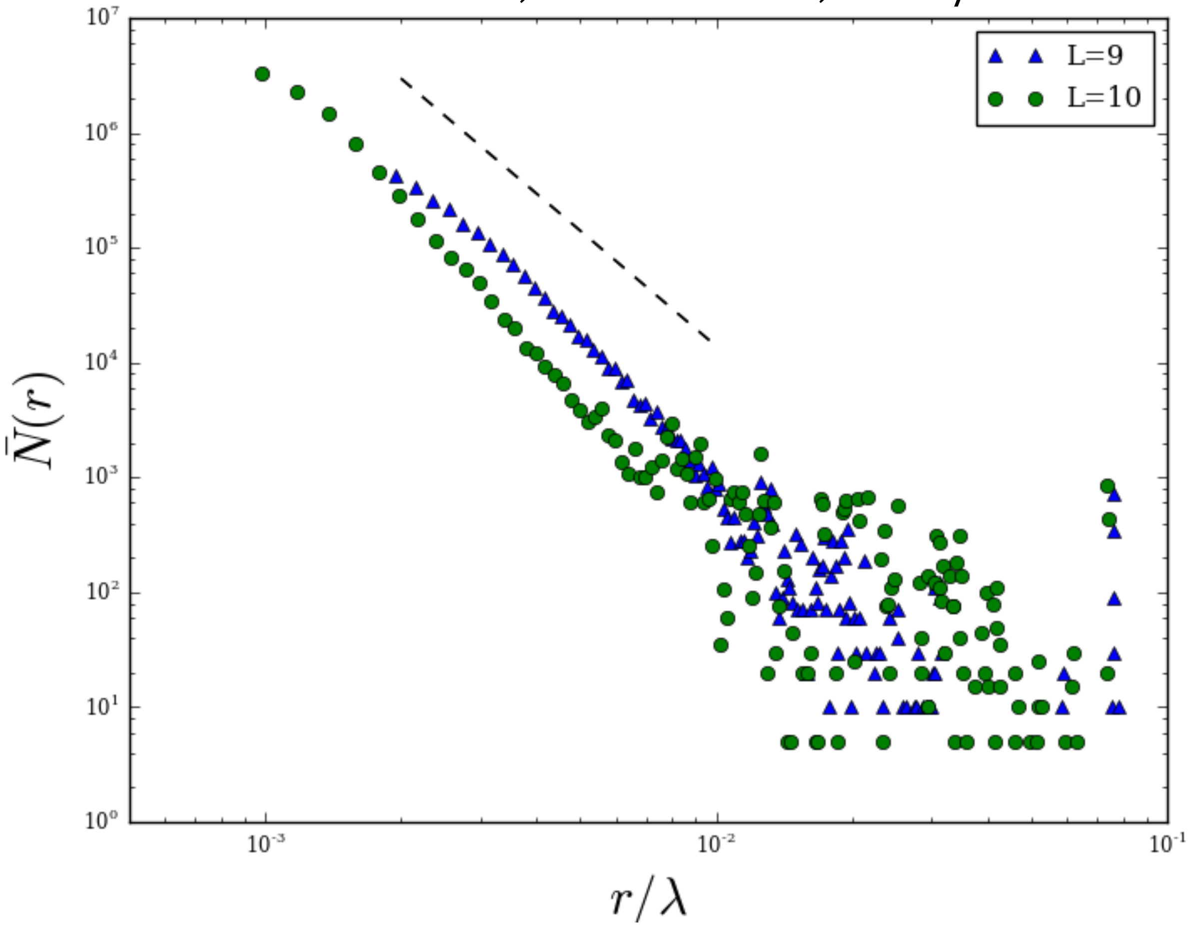


Bubble size distribution

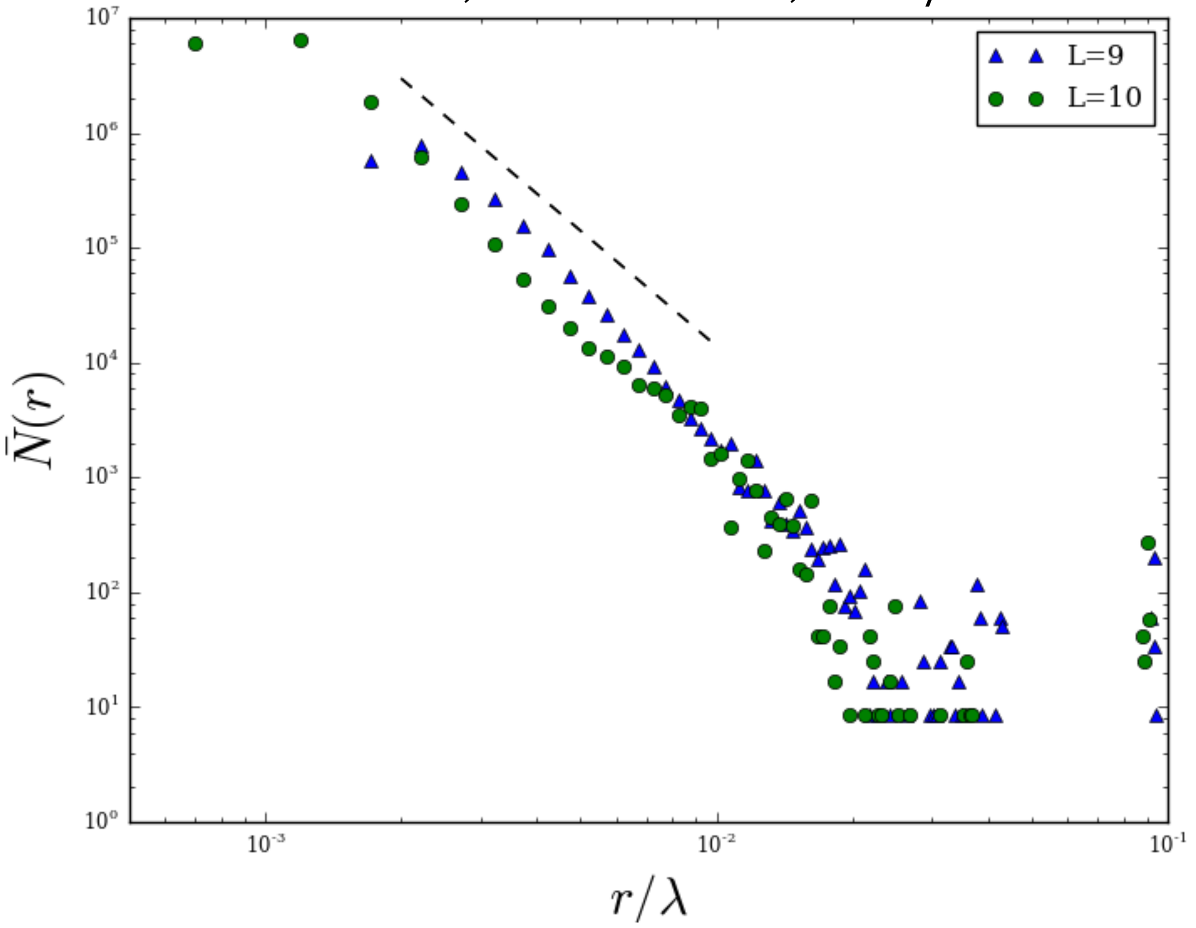
- Averaged $\frac{t}{T} = [0, 2]$
- Grid comparison L9/10

- Bubble size distribution matches -10/3 power-law (Garrett et al 2000)

$Bo = 200, Re = 40\times10^3, L = 9/10$



$Bo = 500, Re = 100\times10^3, L = 9/10$

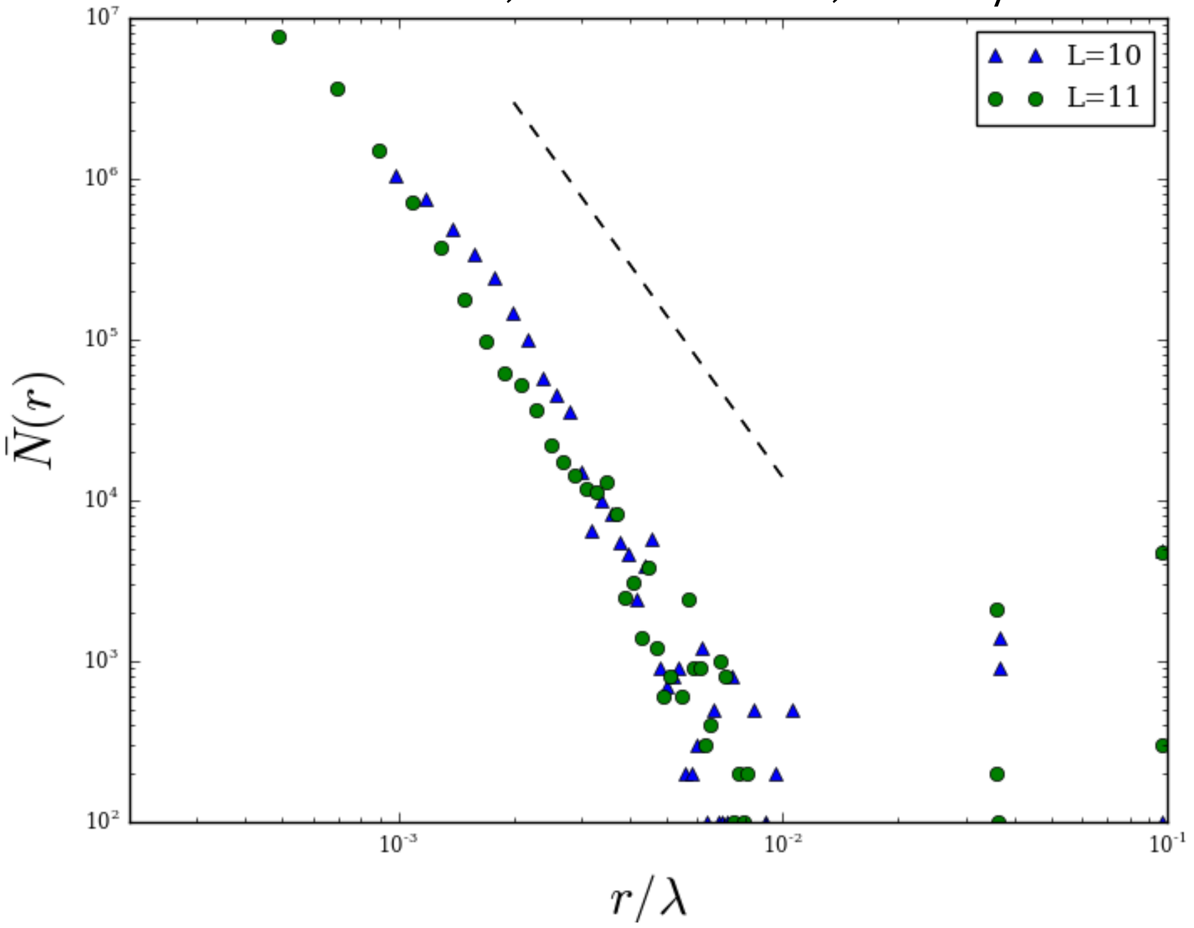


Bubble size distribution

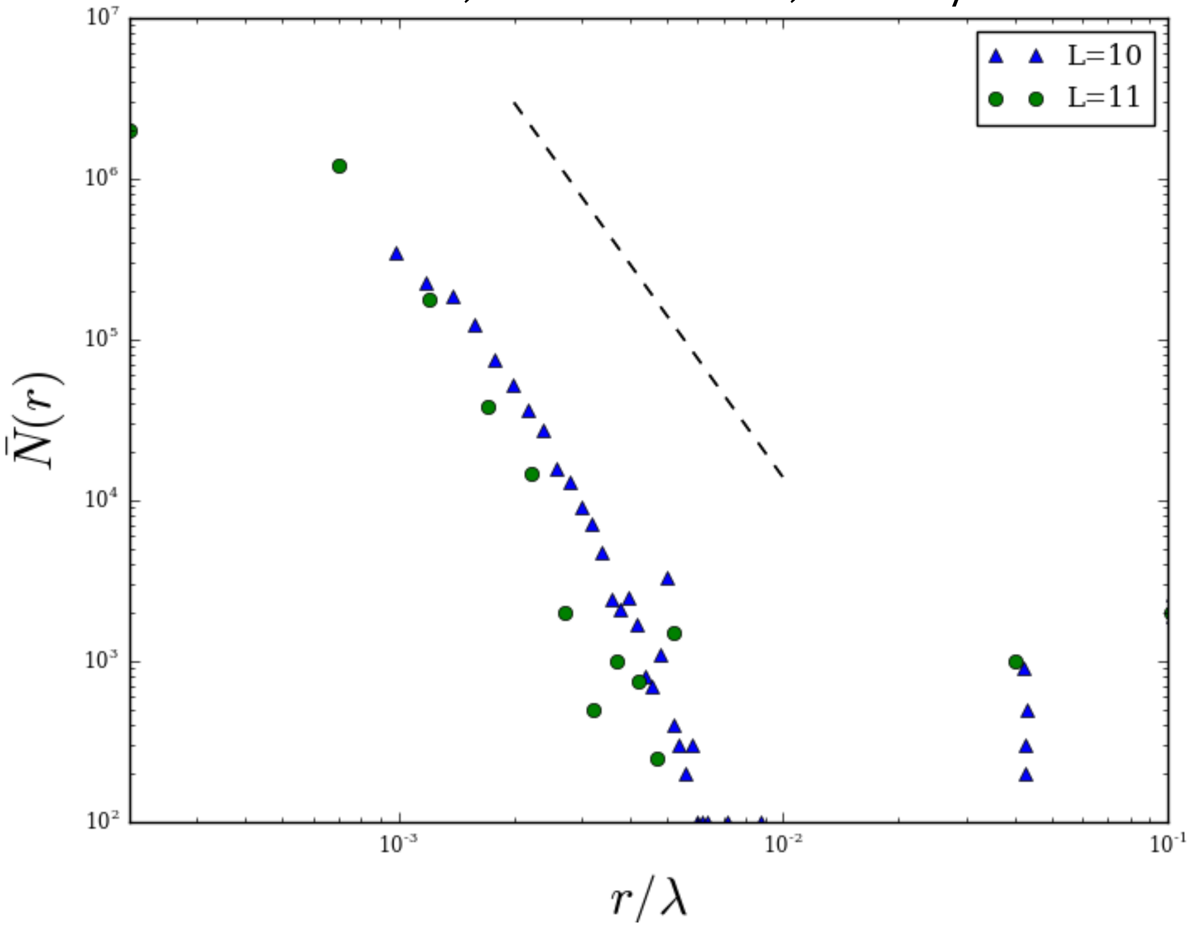
- Averaged $\frac{t}{T} = [0.8, 0.9]$
- Grid comparison 10/11

- Bubble size distribution matches -10/3 power-law (Garrett et al 2000)

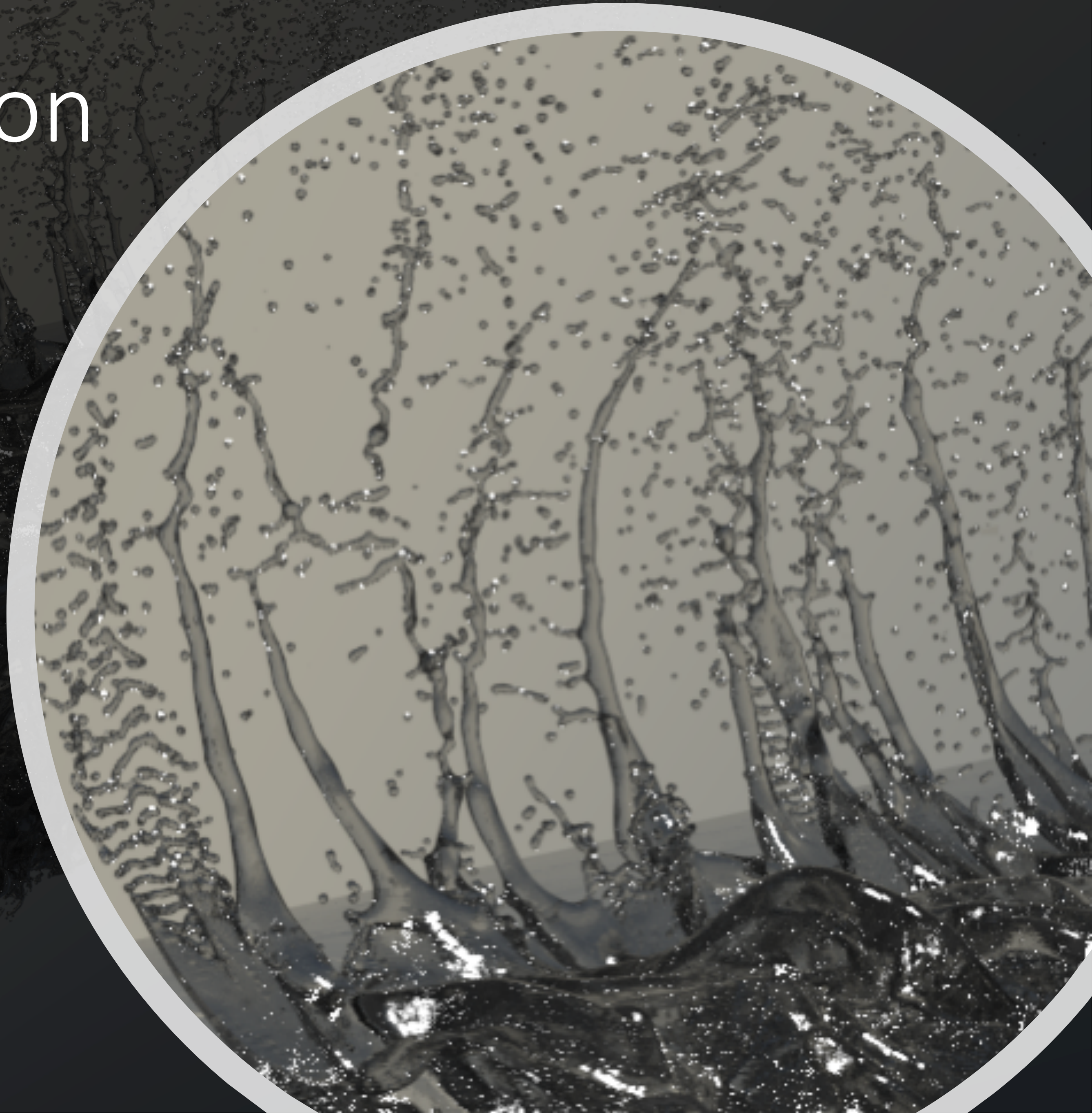
$Bo = 1000, Re = 100\times10^3, L = 10/11$



$Bo = 3000, Re = 200\times10^3, L = 10/11$



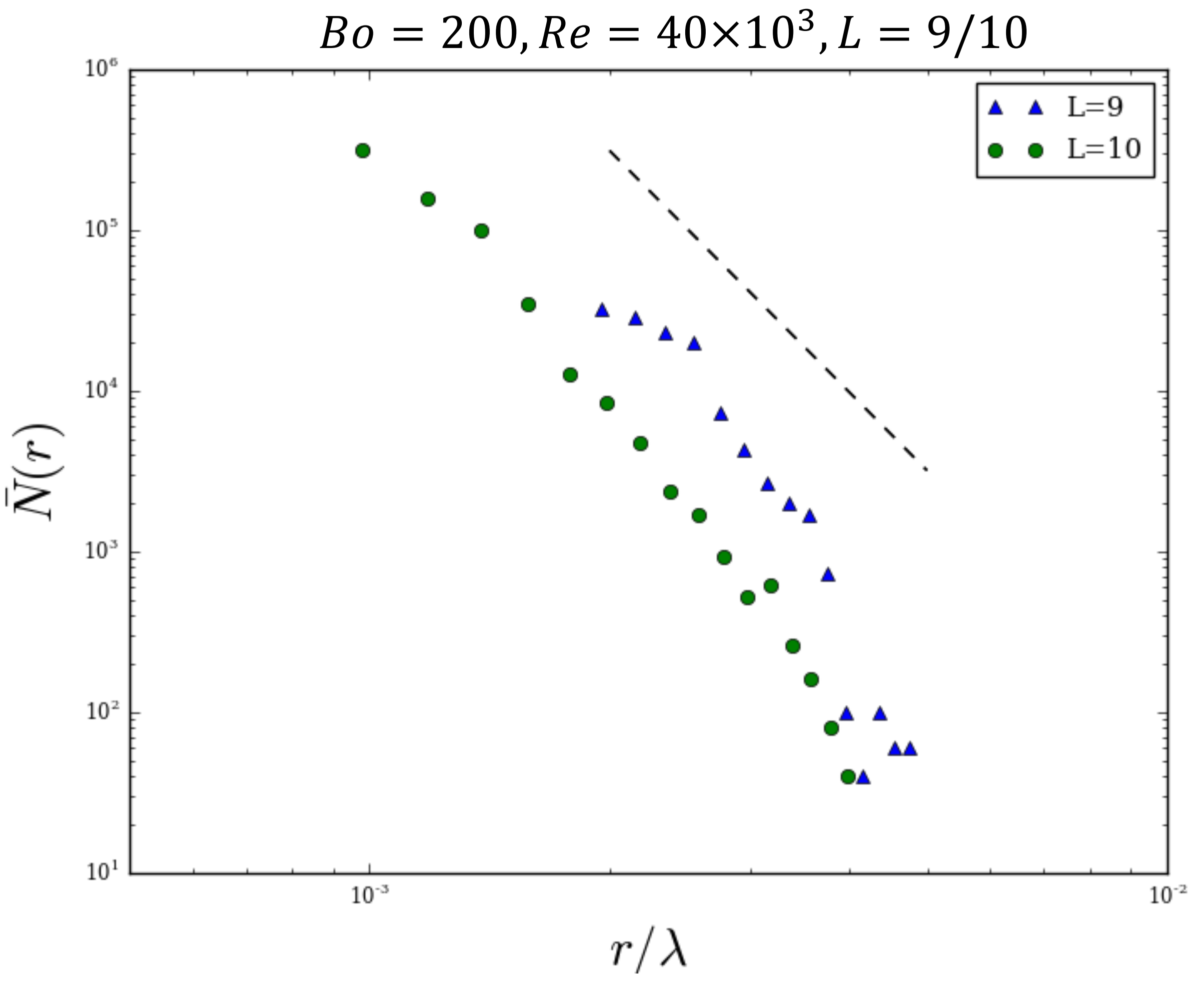
Droplet size distribution

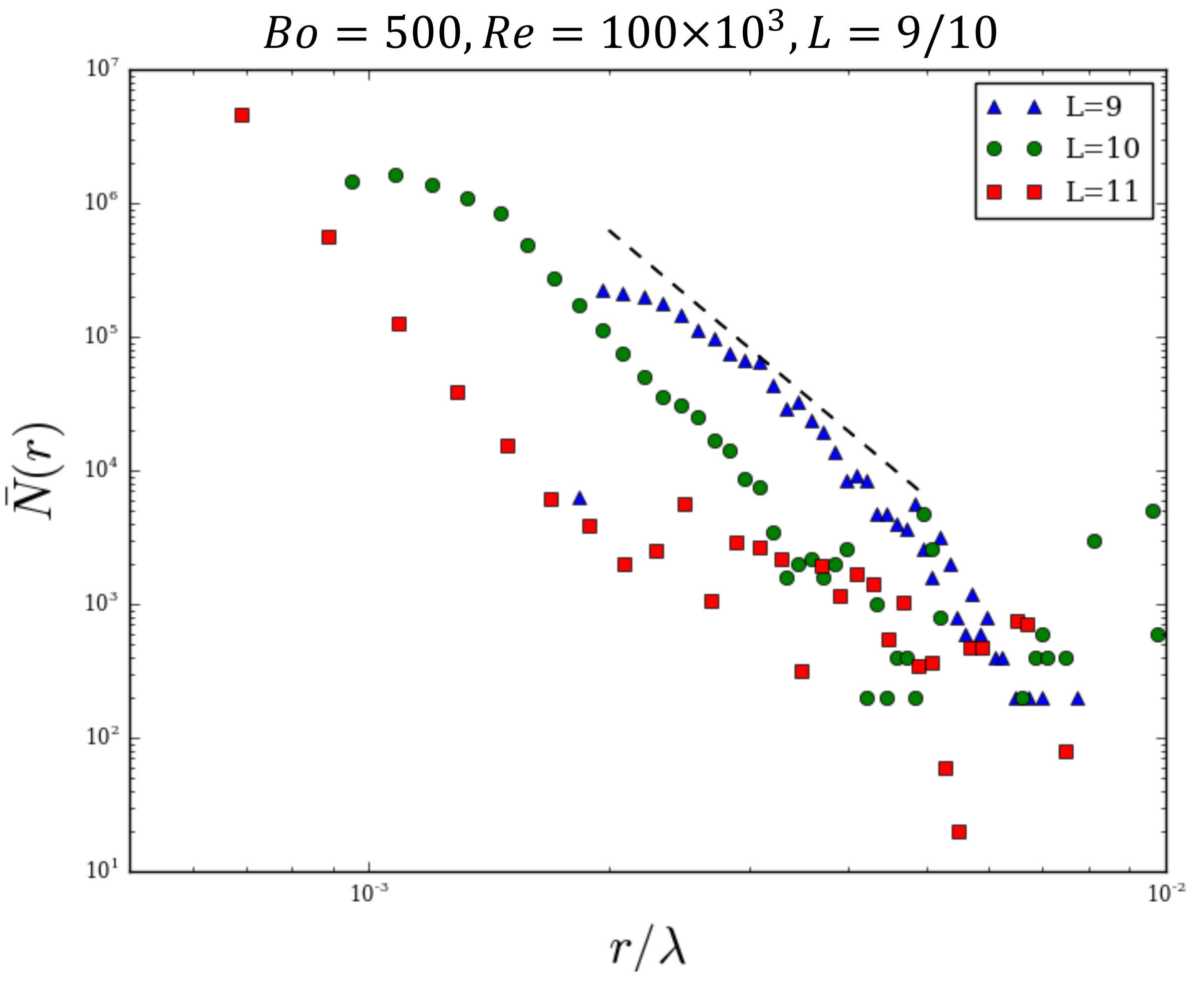


Droplet size distribution

- Averaged $\frac{t}{T} = [0.8, 1.3]$
- Grid comparison L9/10/11

- Bubble size distribution matches approximately -5 power-law
- But **clearly not grid-converged**



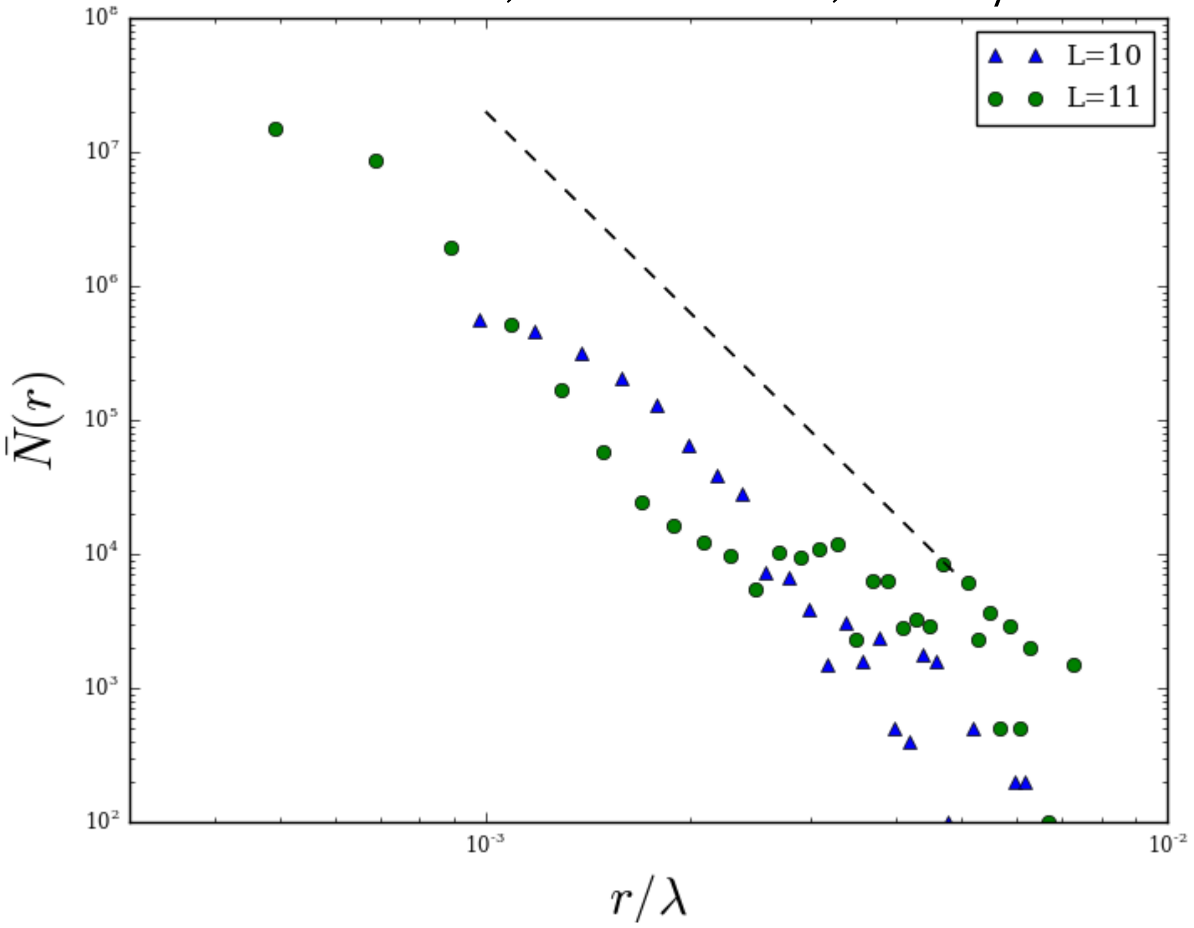


Droplet size distribution

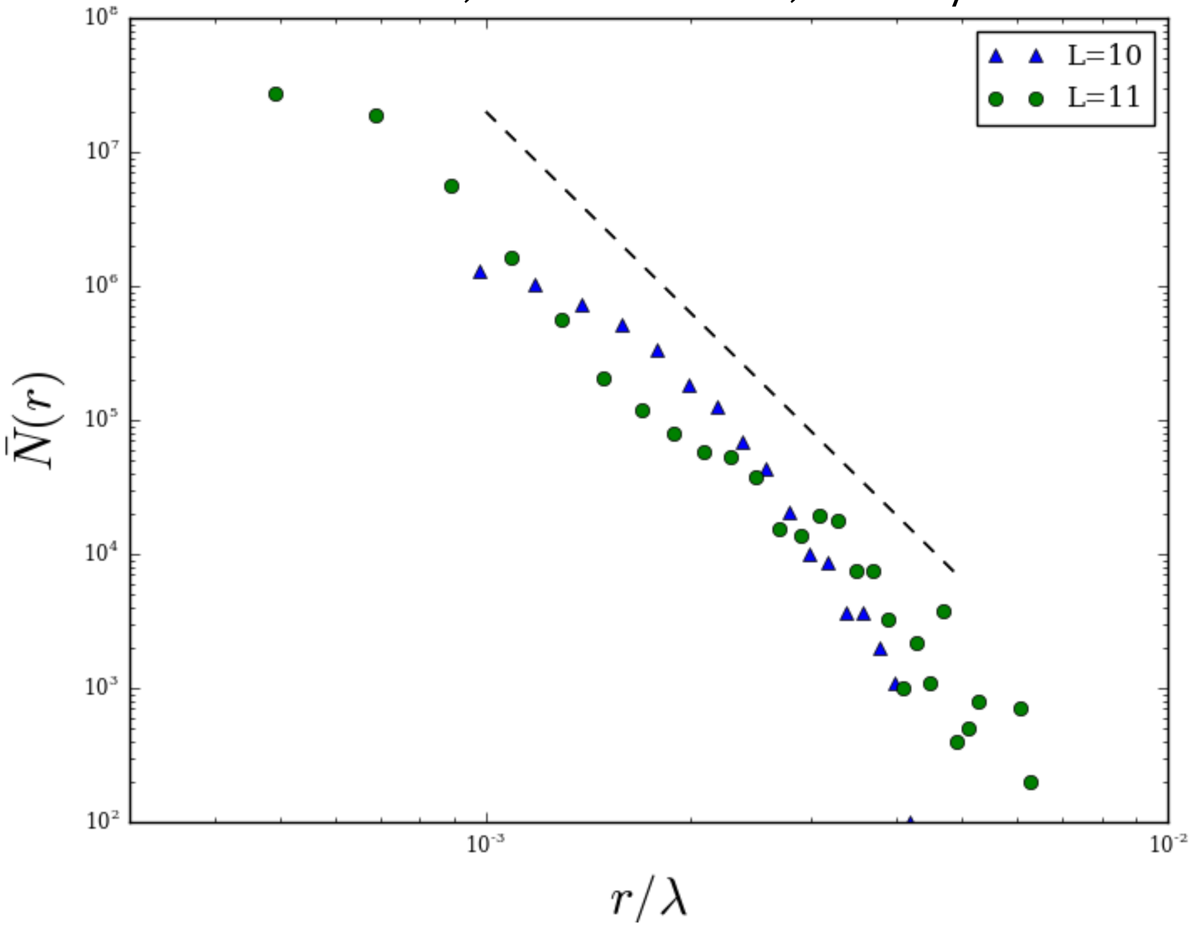
- Averaged $\frac{t}{T} = [0.8, 0.9]$
- Grid comparison 10/11

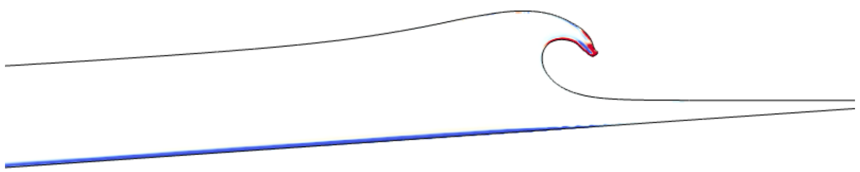
- Bubble size distribution matches approximately -5 power-law
- But **clearly not grid-converged**

$Bo = 1000, Re = 100\times 10^3, L = 10/11$



$Bo = 3000, Re = 200\times 10^3, L = 10/11$





Energetics of solitary waves

Solitary wave shoaling and breaking

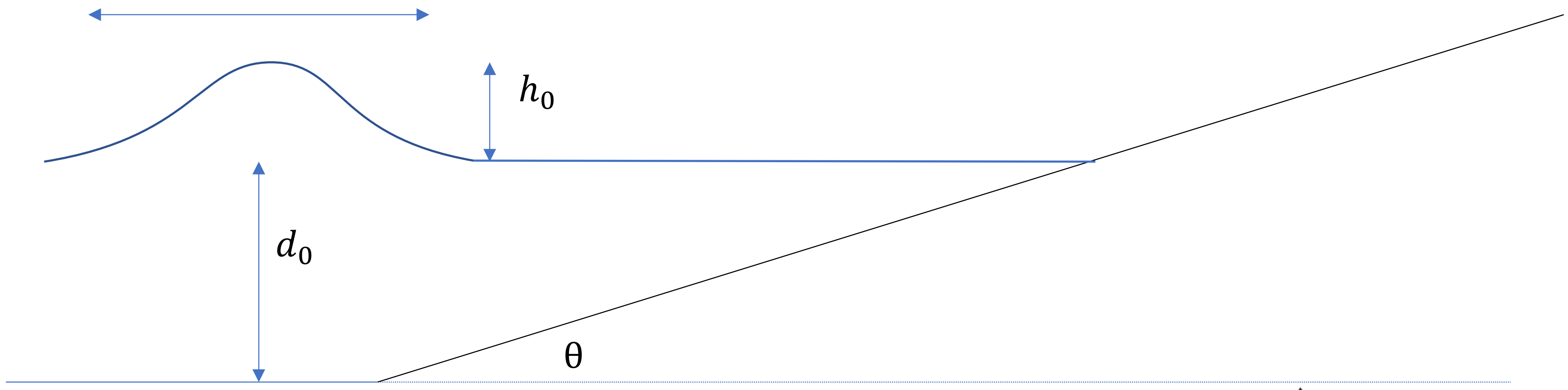
- Consider steady-state offshore solitary wave on approach to beach

$$\eta(x, t = 0) = h_0 \operatorname{sech}^2\left(x\sqrt{\frac{3h_0}{4d_0^3\left(1+\frac{h_0}{d_0}\right)}}\right)$$

Soliton solution to Green-Naghdi equations
Le Metayer et al (2010)

$$c_0 = \sqrt{g d_0\left(1+\frac{h_0}{d_0}\right)}$$

L_0



PRINCETON UNIVERSITY

Solitary wave shoaling and breaking

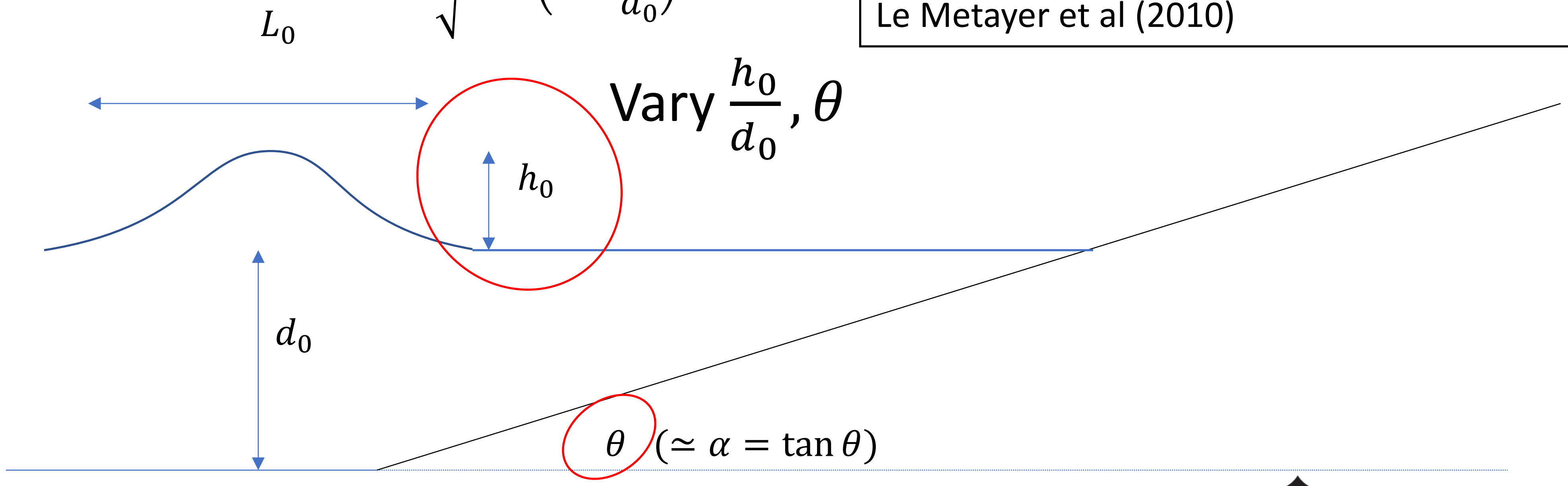
- Consider steady-state offshore solitary wave on approach to beach

$$\eta(x, t = 0) = h_0 \operatorname{sech}^2\left(x\sqrt{\frac{3h_0}{4d_0^3\left(1+\frac{h_0}{d_0}\right)}}\right)$$

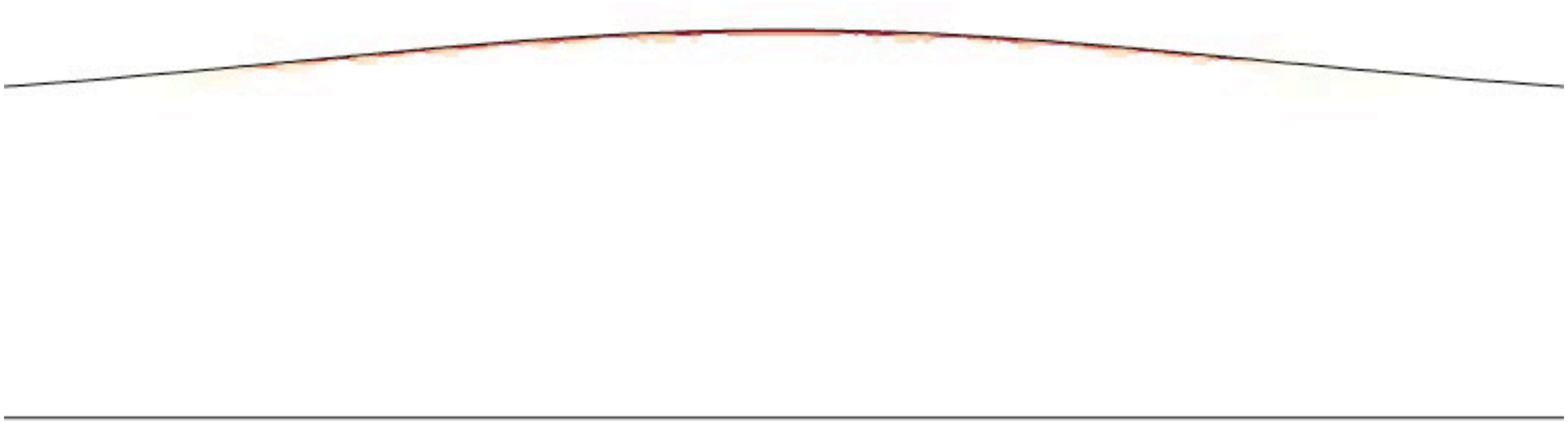
Soliton solution to Green-Naghdi equations
Le Metayer et al (2010)

$$c_0 = \sqrt{gd_0\left(1+\frac{h_0}{d_0}\right)}$$

Vary $\frac{h_0}{d_0}, \theta$



Solitary waves on a beach



Mostert & Deike (2019) J. Fluid Mech. – under review

Spilling breaker

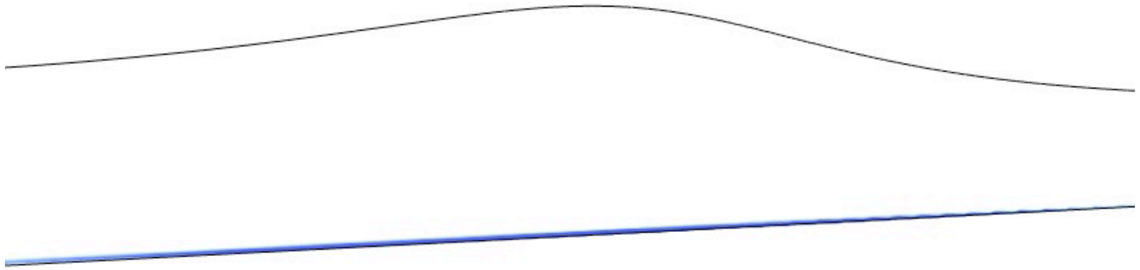
$\alpha = 2^{\circ}, h_0/d_0 = 0.15$

Spilling breaker

$\alpha = 2°, h_0/d_0 = 0.15$

Plunging breaker

$\alpha = 3°, h_0/d_0 = 0.4$

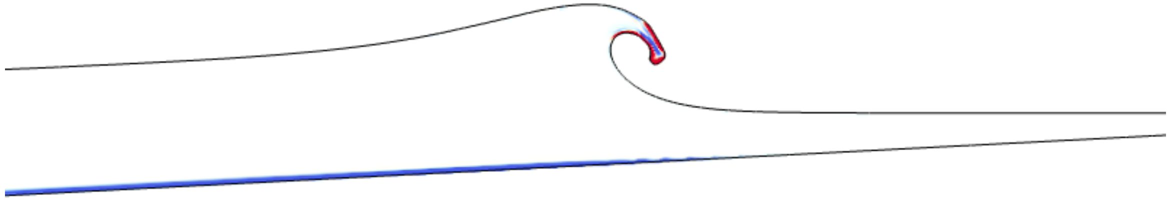


Spilling breaker

$\alpha = 2°, h_0/d_0 = 0.15$

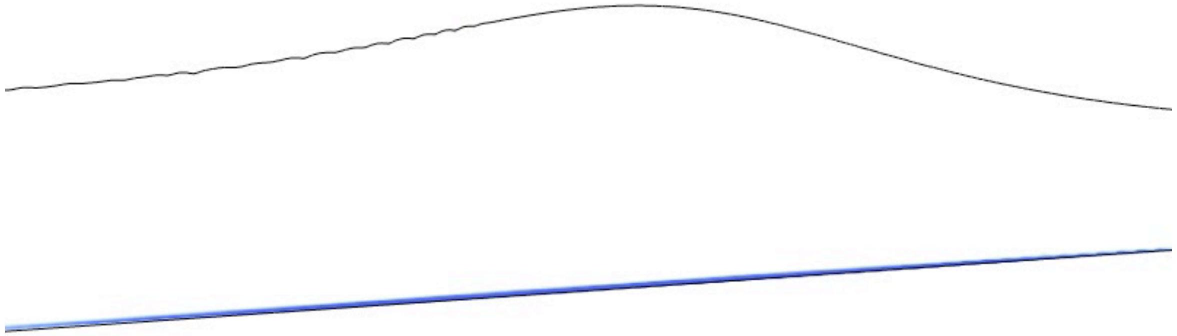
Plunging breaker

$\alpha = 3°, h_0/d_0 = 0.4$



Strong plunging breaker

$\alpha = 4°, h_0/d_0 = 0.5$

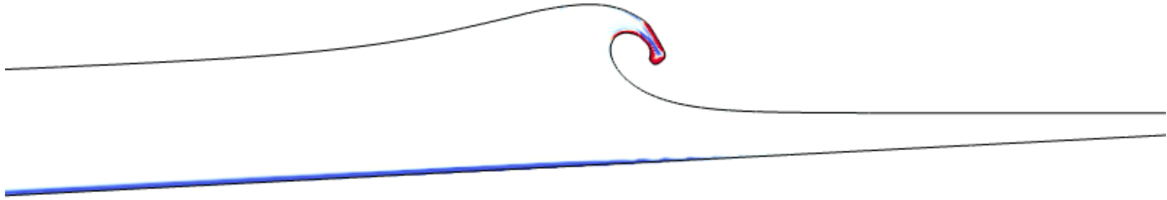


Spilling breaker

$\alpha = 2°, h_0/d_0 = 0.15$

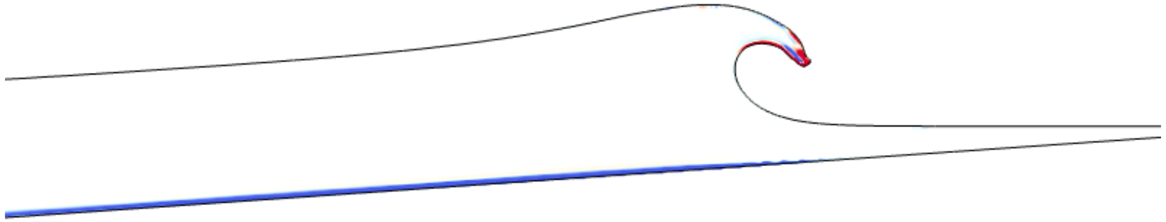
Plunging breaker

$\alpha = 3°, h_0/d_0 = 0.4$



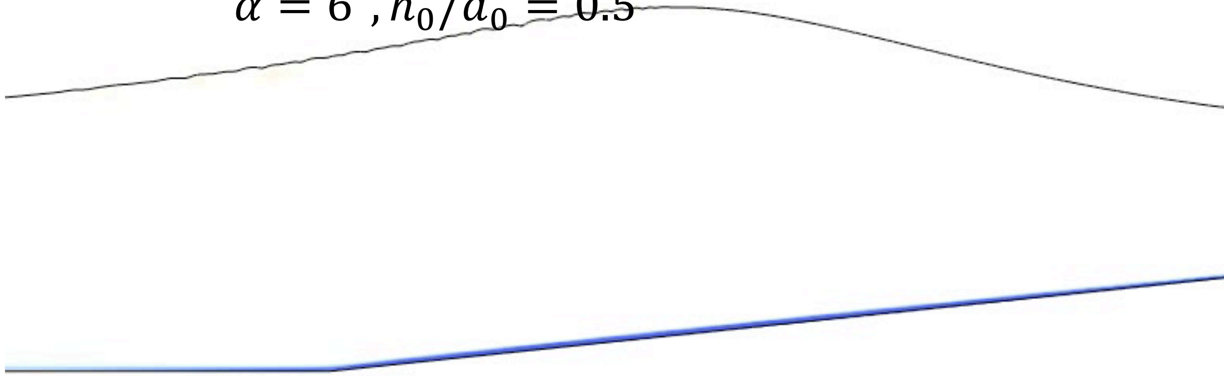
Strong plunging breaker

$\alpha = 4°, h_0/d_0 = 0.5$



Collapsing breaker

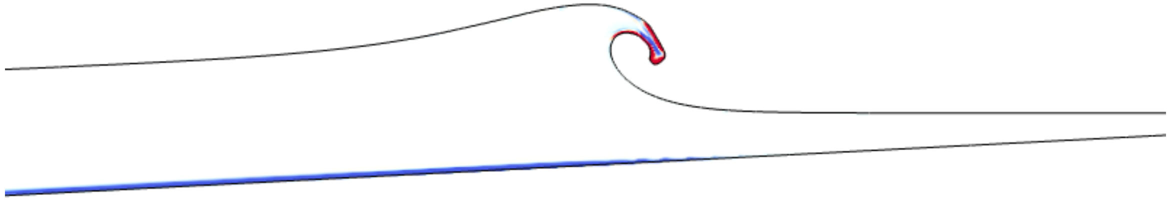
$\alpha = 6°, h_0/d_0 = 0.5$



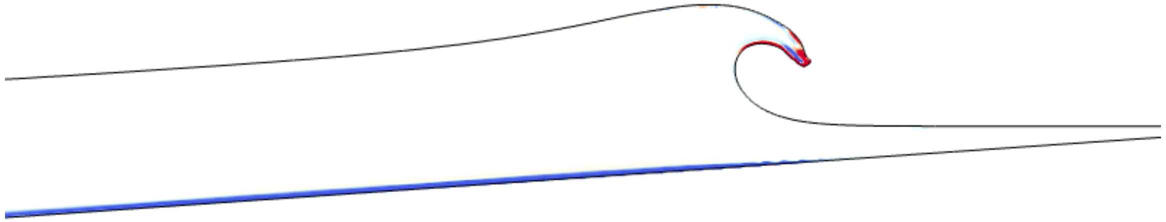
PRINCETON UNIVERSITY

Spilling breaker
$\alpha = 2°, h_0/d_0 = 0.15$

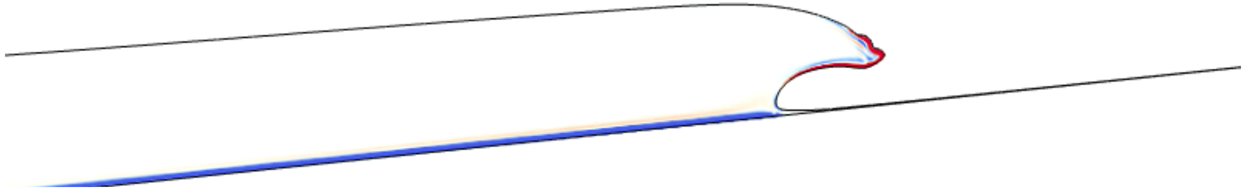
Plunging breaker
$\alpha = 3°, h_0/d_0 = 0.4$



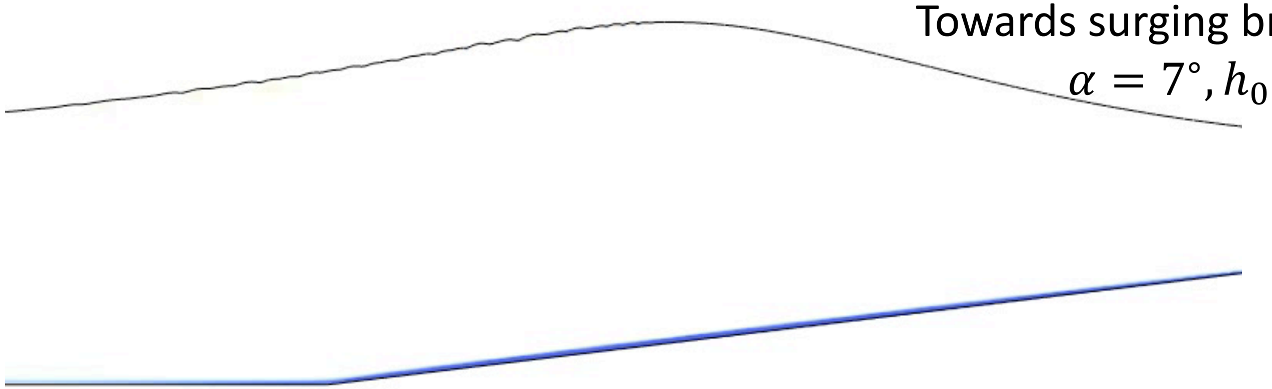
Strong plunging breaker
$\alpha = 4°, h_0/d_0 = 0.5$



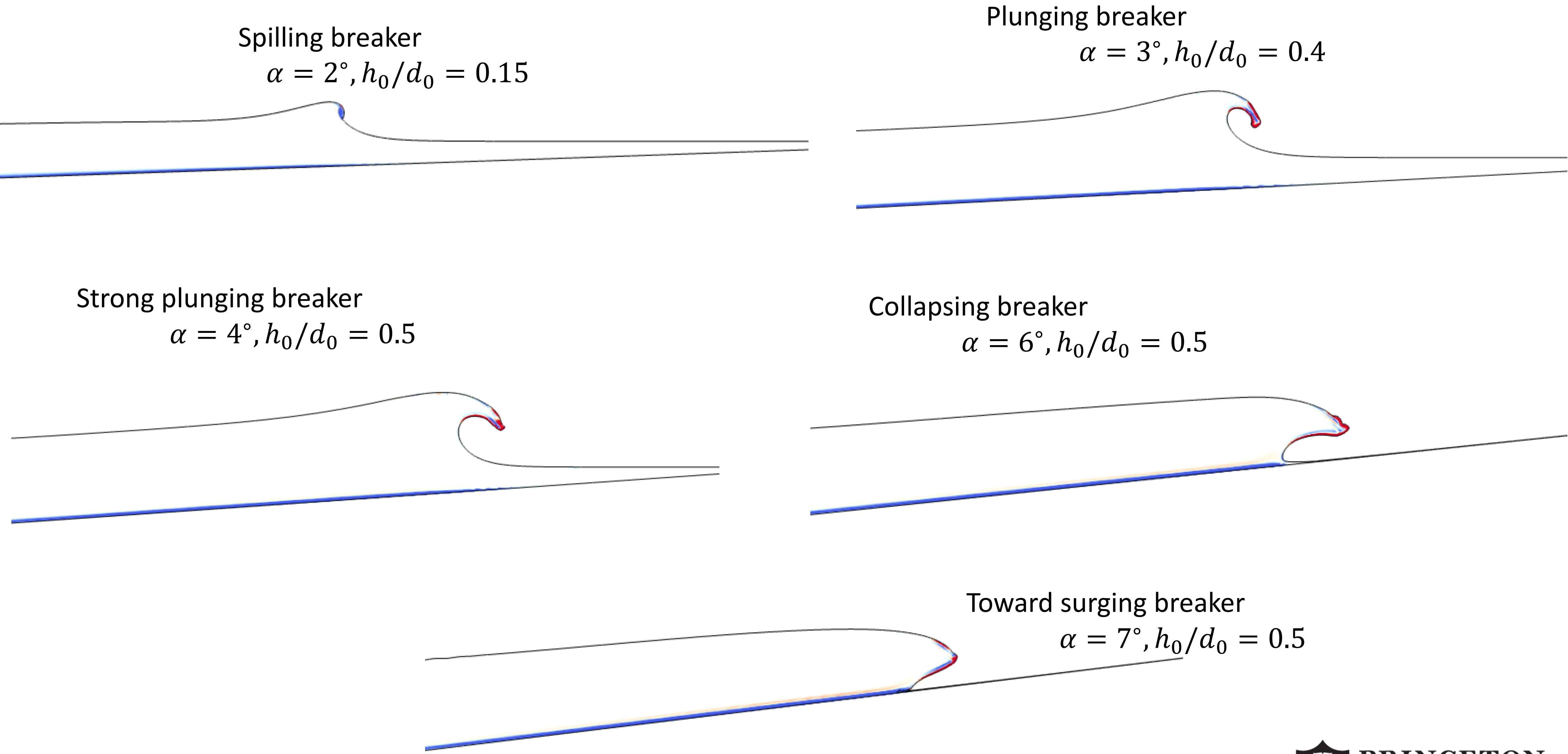
Collapsing breaker
$\alpha = 6°, h_0/d_0 = 0.5$



Towards surging breaker
$\alpha = 7°, h_0/d_0 = 0.5$

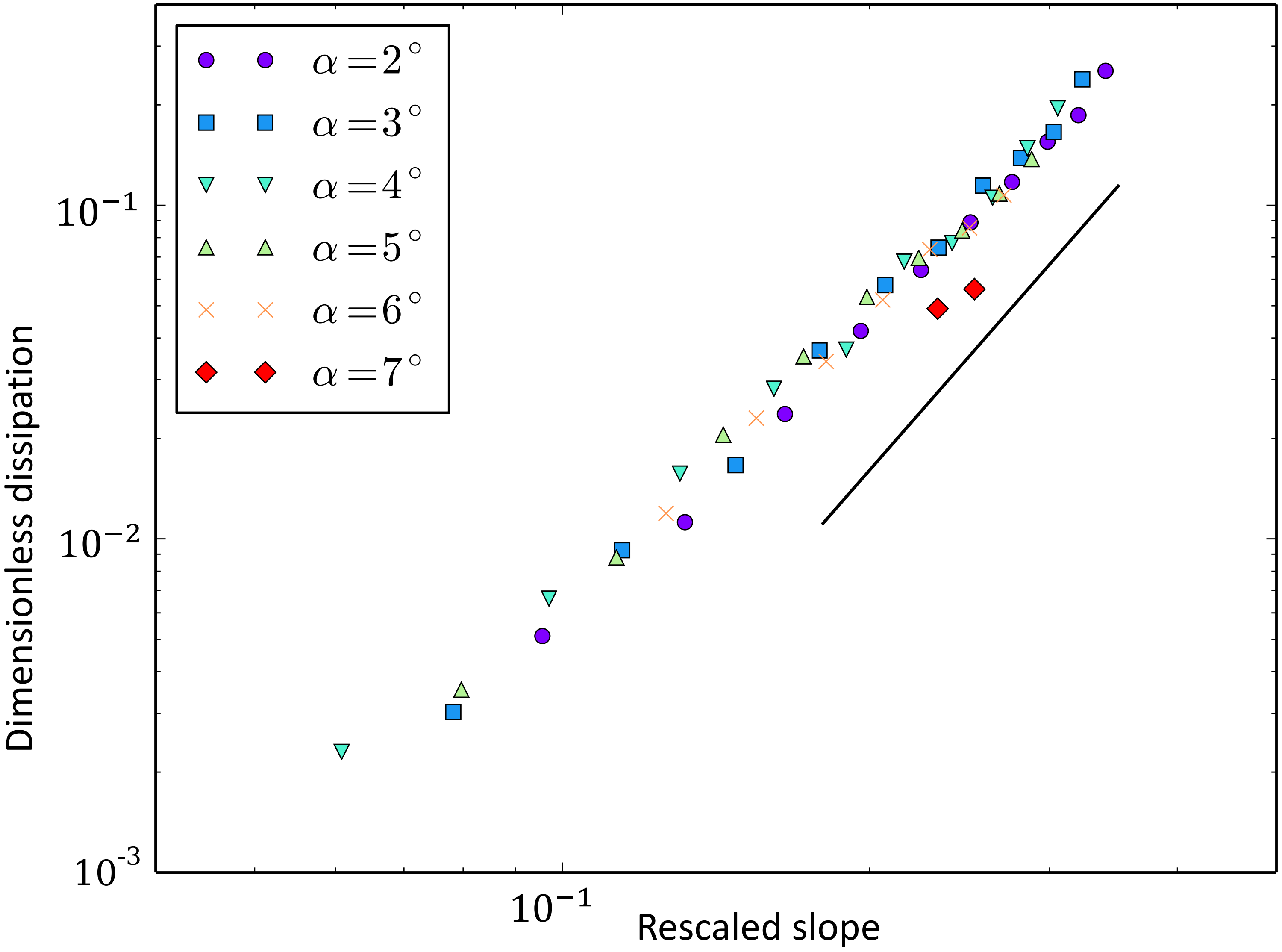


PRINCETON UNIVERSITY



PRINCETON UNIVERSITY

Dissipation vs initial conditions



We can obtain a scaling with initial conditions:

$$\frac{\epsilon_l}{\epsilon_0} \propto \frac{\alpha c^3}{(gh_0)^{3/2}} \left[\frac{gh_0}{c^2} \left(1 - \frac{\alpha c^2}{gh_0} \right) \right]^{\frac{7}{2}}$$

Inertial scaling adapted from deep water (Drazen et al 2008)

Mostert & Deike (2019) J. Fluid Mech. – under review

Acknowledgements

- *John Towns, Timothy Cockerill, Maytal Dahan, Ian Foster, Kelly Gaither, Andrew Grimshaw, Victor Hazlewood, Scott Lathrop, Dave Lifka, Gregory D. Peterson, Ralph Roskies, J. Ray Scott, Nancy Wilkins-Diehr, "XSEDE: Accelerating Scientific Discovery", Computing in Science & Engineering, vol.16, no. 5, pp. 62-74, Sept.-Oct. 2014, doi:10.1109/MCSE.2014.80*
- *Some simulations performed on XSEDE Stampede2 through TG-OCE180010*
- *This work used the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by National Science Foundation grant number ACI-1548562.*
- *Stephane Popinet's Basilisk package – basilisk.fr*